

# Thermal Duality and Non-Singular Cosmology in $d$ -dimensional Superstrings\*

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## Abstract

We are presenting the basic ingredients of a stringy mechanism able to resolve both the Hagedorn instabilities of finite temperature superstrings as well as the initial singularity of the induced cosmology in arbitrary dimensions. These are shown to be generic in a large class of  $(4,0)$  type II superstring vacua, where non-trivial “gravito-magnetic” fluxes lift the Hagedorn instabilities of the thermal ensemble and the temperature duality symmetry is restored. This symmetry implies a universal maximal critical temperature. In all such models there are three characteristic regimes, each with a distinct effective field theory description: Two dual asymptotically cold regimes associated with the light thermal momentum and light thermal winding states, and the intermediate regime where additional massless thermal states appear. The partition function exhibits a conical structure as a function of the thermal modulus, irrespectively of the spacetime dimension. Thanks to asymptotic right-moving supersymmetry, the genus-1 partition function is well-approximated by that of massless thermal radiation in all of the three effective field theory regimes. The resulting time-evolution describes a bouncing cosmology connecting, via spacelike branes, a contracting thermal “winding” Universe to an expanding thermal “momentum” Universe, free of any essential curvature singularities. The string coupling remains perturbative throughout the cosmological evolution. Bouncing cosmologies are presented for both zero and negative spatial curvature.

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# 1 Introduction

Observational evidence strongly supports that during an early cosmological era, the matter content of the Universe was in (near) thermal equilibrium, with very high temperature. If the degrees of freedom are to be described by a set of local quantum fields, such a state results in a singular cosmology. Indeed, if we follow the cosmological evolution backward in time, using Einstein's gravity field equations, we are driven to the initial curvature singularity [1]. Even if a period of inflation preceded the high temperature phase, it is found in typical field theory models that the cosmological evolution begins at a singularity.

In string theory we expect a drastically different picture to emerge since new purely stringy degrees of freedom can dominate the high curvature and high temperature regimes, leading to phenomena that do not admit a conventional field theory description [2], with Riemannian concepts breaking down. String oscillators and winding states become relevant around the Hagedorn temperature  $T_H$  (which is of order the string scale  $M_s$ ), before the onset of curvature singularities, and drive a phase transition towards a new stringy thermal vacuum [3–12]. The simplest way to isolate the relevant critical phenomena is via the Euclidean description of the thermal system, where Euclidean time is compactified on a circle with period given by the inverse temperature [5–7]. At temperatures just above Hagedorn, certain string states winding the Euclidean time circle become tachyonic. These instabilities can be lifted either by condensing the tachyons [5, 7], or by turning on special gravito-magnetic fluxes, which inject into the thermal vacuum non-trivial winding and momentum charges, as in [13–16]. If a stable stringy phase gets realized, it could be that the back-reacted cosmological evolution is non-singular and the initial singularity is absent.

A mechanism within which the Hagedorn instabilities of the string gas are resolved and the initial curvature singularity is bypassed was realized recently in a class of two-dimensional superstring cosmologies, the so called Hybrid cosmologies [16]. The scope of the present work is to show that the key ingredients of this mechanism are generic in a diverse class of higher dimensional superstring models as well. In all of these models, finite temperature is introduced along with non-trivial gravito-magnetic fluxes [13–16], which lead to a restoration of the thermal duality symmetry of the partition function:  $Z(\beta/\beta_c) = Z(\beta_c/\beta)$ . Here  $\beta$  denotes the period of the Euclidean time cycle, attaining a critical value  $\beta_c$  at the self-dual point. At this critical point additional massless thermal states appear, enhancing the local

Euclidean gauge symmetry. Typical examples include the tachyon-free type II  $\mathcal{N}_4 = (4, 0)$  models at finite temperature and in the presence of non-trivial gravito-magnetic fluxes, which are described in great detail in the literature [13–16]. The fundamental properties of these models, which can lead to the resolution of the Hagedorn and the initial singularity, are well understood from the recent study of the two dimensional Hybrid models [16], and are exhibited below:

- The canonical thermal ensemble is modified by turning on non-trivial gravito-magnetic fluxes, which lift the usual Hagedorn instabilities. The fluxes inject non-trivial winding and momentum charges into the thermal vacuum and render the mass of the would-be tachyonic states semi-positive definite. The tachyon-free models are equivalent to freely acting asymmetric orbifolds obtained by modding out with  $(-1)^{F_L} \delta_0$ ,  $F_L$  being the left-moving space-time fermion number and  $\delta_0$  an order-2 shift along the Euclidean time cycle. Essentially, the fluxes regulate the contribution to the free energy of the massive string states.
- Not only is the resulting spectrum of thermal masses semi-positive definite, but also the partition function is duality invariant under  $\beta \rightarrow \beta_c^2/\beta$ , and it is finite for all values of  $\beta$ . At the critical point, new massless thermal states appear extending the  $U(1)_L$  gauge symmetry associated to the Euclidean time cycle to a non-Abelian  $[SU(2)_L]_{k=2}$  symmetry. This is a purely stringy phenomenon, absent in any conventional field theory model. The self-dual point  $\beta = \beta_c$  is realized at the so called fermionic point. This universal property of all such superstring models follows from the conformal transformation properties of the left-moving  $NS$  vacuum ( $h_L = -\frac{1}{2}$ ).
- The extra massless states at the critical (fermionic) point have non-trivial momentum and winding charges so that  $p_L = \pm 1$  and  $p_R = 0$ . These two extra states together with the thermal radius modulus give rise to the  $SU(2)$  enhanced symmetry. At the critical point, the massless states give rise to non-trivial backgrounds which admit a localized brane interpretation in the Euclidean.
- For  $\beta/\beta_c \gg 1$ , the asymptotic behavior of the thermal partition function is dominated by the light thermal momentum states giving rise to the characteristic behavior of massless thermal radiation in  $d$  dimensions, modulo exponentially suppressed contri-

butions from the massive string oscillator states:

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left( \frac{\beta_c}{\beta} \right)^{d-1} + \mathcal{O}(e^{-\beta/\beta_c}), \quad (1.1)$$

where  $n^*$  counts the number of effectively massless degrees of freedom;  $\Sigma_d$  is the Stefan-Boltzmann constant for radiation and  $V_{d-1}$  is the spatial volume. What is extremely important is that thanks to the thermal duality symmetry, the asymptotic behavior for  $\beta/\beta_c \ll 1$  is dual-to-thermal, as it is dominated by the light thermal winding states:

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left( \frac{\beta}{\beta_c} \right)^{d-1} + \mathcal{O}(e^{-\beta_c/\beta}). \quad (1.2)$$

Here also, the oscillator states give exponentially suppressed contributions. The contribution of the massive oscillator states remains finite at the critical point, as the fluxes modify and effectively reduce the density of thermally excited massive oscillator states. In most cases, the contribution of the massive oscillator states never dominates over the thermally excited massless states due to asymptotic supersymmetry [13–18].

- This behavior indicates the appropriate, *duality invariant definition of the temperature*  $T$ , valid in both asymptotic thermal regimes. Defining the thermal modulus  $\sigma$  by  $e^\sigma = \beta/\beta_c$ , the duality invariant temperature is given by  $T \equiv T_c e^{-|\sigma|}$ . Thus the temperature in these configurations, and consequently the energy density and pressure, never exceed a critical value. The maximal critical temperature is given by  $T_c = 1/\beta_c$ . In both asymptotic regimes ( $T \ll T_c$ ), the partition function can be expressed in terms of the self-dual temperature as follows:

$$\frac{Z}{V_{d-1}} = n^* \Sigma_d T_c^{d-1} \left( \frac{T}{T_c} \right)^{d-1} + \mathcal{O}(e^{-T_c/T}). \quad (1.3)$$

We conclude that the stringy thermal system has three characteristic regimes: The two dual phases of light thermal momenta and light thermal windings, and a third intermediate regime corresponding to the extended symmetry point, where vortices described by massless thermal states carrying non-trivial momentum and winding charges become relevant.

The presence of the localized massless states is crucial since *they can marginally induce transitions between purely momentum and purely winding states*, thus driving a phase transition between the two asymptotic regimes. As in [16], this phase transition admits a geometrical description, in terms of a “T-fold”, with branes localized at the critical point

gluing the “momentum” and “winding” spaces. This gluing mechanism was explicitly realized in the two dimensional Hybrid model [15, 16], where the partition function and its conical structure were determined beyond any  $\alpha'$  approximation at the perturbative genus-1 level. In the Hybrid model, the ingredients described above not only treat successfully the Hagedorn transition, but also they lead to non-singular thermal cosmologies in contrast to field theoretic cases. In this work, utilizing the fundamental ingredients and especially the branes sourced by the extra massless thermal states, we show that non-singular string cosmologies also exist in higher dimensions.

The plan of the paper is as follows. In section 2, we construct type II thermal vacua in arbitrary dimension  $d \geq 2$ , which are free of Hagedorn instabilities due to the presence of certain gravito-magnetic fluxes. We show that up to exponentially suppressed terms, the corresponding partition functions are well-approximated by the contributions of the thermally excited massless states up to the critical point. In section 3, we present an effective action valid in all three regimes associated to the “thermal momentum phase” ( $\beta > \beta_c$ ), the “thermal winding phase” ( $\beta < \beta_c$ ) and the non-geometrical  $[SU(2)_L]_{k=2}$  point ( $\beta = \beta_c$ ). At the critical temperature, additional massless thermal states source negative pressure contributions to the effective action. Cosmological solutions free of initial singularities in arbitrary dimension are exhibited in section 4. They are compatible with perturbation theory throughout the evolution and describe bouncing cosmologies, where a phase transition connects a contracting thermal winding space-time to an expanding thermal momentum space-time. Solutions which are radiation or curvature dominated at both the very early and very late cosmological times are presented. Finally, our results and perspectives are summarized in section 5.

## 2 Thermal duality and the Hagedorn transition

In this section we construct tachyon-free thermal configurations, starting with type II  $\mathcal{N}_4 = (4, 0)$  models in various dimensions. The left-moving worldsheet degrees of freedom give rise to 16 real supercharges, while the remaining right-moving supersymmetries are broken spontaneously via asymmetric geometrical fluxes [13–16]. Geometrical fluxes generalize the Scherk-Schwarz mechanism [19] to string theory [6, 20, 21]. At certain points in moduli space, where the moduli participating in the breaking of the right-moving supersymmetries

attain values close to the string scale, the local gauge symmetry is enhanced to a non-Abelian one [13–16]. Finite temperature and quantum effects will stabilize these moduli at such extended symmetry points [22–24]. Interesting examples include the two-dimensional Hybrid models, where the right-moving sector is characterized by unbroken massive spectrum (boson/fermion) degeneracy symmetry (*MSDS*) [14–16].

Finite temperature is introduced along with non-trivial gravito-magnetic fluxes, threading the Euclidean time cycle together with other cycles responsible for the breaking of the right-moving supersymmetries [13–16]. These fluxes inject into the thermal vacuum non-trivial momentum and winding charges and lift the Hagedorn instabilities of the thermal ensemble. To see how this occurs, recall that for special values of the fluxes, the model is equivalent to a freely acting asymmetric orbifold of the form  $(-1)^{F_L}\delta_0$ , where  $F_L$  is the left-moving space-time fermion number and  $\delta_0$  is a  $Z_2$ -shift along the Euclidean time circle [13]. The genus-1 partition function is given by

$$Z = \frac{V_{d-1}}{(2\pi)^{d-1}} \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2^{(d+1)/2}} \frac{1}{(\eta\bar{\eta})^8} \frac{1}{2} \sum_{\bar{a},\bar{b}} (-1)^{\bar{a}+\bar{b}} \frac{\bar{\theta}[\frac{\bar{a}}{\bar{b}}]^4}{\bar{\eta}^4} \Gamma_{(10-d,10-d)[\frac{\bar{a}}{\bar{b}}]} \\ \times \sum_{m_0,n_0} \left( V_8 \Gamma_{m_0,2n_0}(R_0) + O_8 \Gamma_{m_0+\frac{1}{2},2n_0+1}(R_0) - S_8 \Gamma_{m_0+\frac{1}{2},2n_0}(R_0) - C_8 \Gamma_{m_0,2n_0+1}(R_0) \right), \quad (2.1)$$

where  $V_{d-1}$  stands for the volume of the large spatial directions. Here  $\Gamma_{(10-d,10-d)[\frac{\bar{a}}{\bar{b}}]}$  denotes the asymmetrically half-shifted lattice associated with the compact directions, leading to the breaking of the right-moving supersymmetries at zero temperature. The last line in the integrand gives the combined effect of finite temperature and the gravito-magnetic fluxes, amounting to a thermal action on the left-movers of the worldsheet. As a result, the orbifold deviates from the conventional thermal one in sectors with odd right-moving fermion number  $F_R$ . At the preferred right-moving extended symmetry points, these sectors are massive with masses of order the string scale. Sectors with even  $F_R$  and in particular the initially massless bosons and fermions are thermally excited as in the canonical ensemble. Thus the deformed model does not differ appreciably from the conventional thermal one at temperatures lower than Hagedorn. The presence of the gravito-magnetic fluxes however allows for the existence of stable phases above the Hagedorn temperature for which the canonical thermal ensemble fails to converge.

The left-moving  $O_8$  sector carries non-trivial momentum and winding charges so that

$$\frac{1}{2} \min |p_L^2 - p_R^2| = \min \left| \left(m_0 + \frac{1}{2}\right)(2n_0 + 1) \right| = \frac{1}{2}. \quad (2.2)$$

This is just enough to produce a state that is at least holomorphically or anti-holomorphically massless, ensuring the absence of tachyons in the spectrum of thermal masses for all values of the thermal modulus  $R_0$ . In addition, the model remains tachyon free under all marginal deformations of the transverse dynamical moduli associated with the compact manifold. The partition function is finite and invariant under thermal duality symmetry,  $R_0 \rightarrow 1/(2R_0)$  (together with the  $S_8 \leftrightarrow C_8$  interchange), with the self-dual critical point occurring at the fermionic point  $R_c = 1/\sqrt{2}$ . At  $R_0 = R_c$ , additional thermal states become massless, enhancing the  $U(1)_L$  gauge symmetry associated with the compact Euclidean time circle to a non-Abelian  $[SU(2)_L]_{k=2}$  symmetry. Their masses are equal to

$$m^2 = \left( \frac{1}{2R_0} - R_0 \right)^2. \quad (2.3)$$

At the fermionic point, the corresponding left- and right-moving momenta and the associated vertex operators are given by

$$p_L = \pm 1, \quad p_R = 0, \quad O_{\pm} = \psi_L^0 e^{\pm iX_L^0} O_R, \quad (2.4)$$

where  $O_R$  are weight  $(0, 1)$  right-moving operators. Such states can marginally induce transitions between purely thermal winding and purely thermal momentum states, and in addition exchange the spinor chirality,  $S_8 \leftrightarrow C_8$  [16]. As we will see, these states induce *a universal, non-analytic conical structure in the partition function  $Z$*  as a function of the thermal modulus  $R_0$ , irrespectively of the dimensionality of the model.

Thermal duality implies a maximal critical temperature and the existence of two dual asymptotically cold regimes dominated by the light thermal momenta and the light thermal windings respectively. The regime of light momenta,  $R_0 \gg R_c$ , gives rise to a thermal phase with temperature  $T = 1/\beta$ , where  $\beta = 2\pi R_0$  is the period of the Euclidean time circle. In the regime of light windings,  $R_0 \ll R_c$ , the vortices can be interpreted by T-duality as ordinary thermal excitations associated with a large circle of period  $\tilde{\beta} = \beta_c^2/\beta$ . The corresponding temperature is given by  $T = 1/\tilde{\beta} = \beta/\beta_c^2$ . Thus the system at small radii is again effectively cold. The two phases are distinguished by the light thermally excited spinors: At large radii these transform under the  $S_8$ -Spinor of  $SO(8)$ , while at small radii they transform

under the conjugate  $C_8$ -Spinor. At  $R_0 = R_c$ , we find the intermediate regime where extra massless thermal states appear and enhance the local gauge symmetry to a non-Abelian gauge symmetry. As in Ref. [16], the extra massless thermal states give rise to genus-0 backgrounds, present when  $R_0 = R_c$ , admitting a Euclidean brane interpretation [16]. Since transitions between purely momentum and purely winding states can occur in the presence of such condensates, the branes “glue together” the light momentum and light winding regimes.

Defining the thermal modulus  $\sigma$  by  $R_0/R_c = e^\sigma$ , the duality invariant expression for the temperature, valid in both the winding and momentum phases, is given by

$$T = T_c e^{-|\sigma|}, \quad T_c = \frac{1}{\beta_c} = \frac{1}{\sqrt{2\pi}}, \quad (2.5)$$

attaining a maximal critical value  $T_c$  at the self-dual point  $\sigma = 0$ . As a result, the energy density and pressure in these models are bounded, never exceeding certain maximal values.

To identify further universal features concerning the thermal effective potential, it is illuminating to compare two models of different dimensionality, where the mechanism leading to the resolution of the Hagedorn singularity is transparent:

- A  $d = 2$  Hybrid model, where the first line of Eq. (2.1) is given by

$$\frac{1}{(\eta\bar{\eta})^8} \frac{1}{2} \sum_{\bar{a}, \bar{b}} (-1)^{\bar{a}+\bar{b}} \frac{\bar{\theta}[\frac{\bar{a}}{\bar{b}}]^4}{\bar{\eta}^4} \Gamma_{(8,8)[\frac{\bar{a}}{\bar{b}}]} = \frac{\Gamma_{E_8}(\tau)}{\eta^8} (\bar{V}_{24} - \bar{S}_{24}), \quad (2.6)$$

exhibiting holomorphic/anti-holomorphic factorization and right-moving *MSDS* structure, as exemplified by the identity  $\bar{V}_{24} - \bar{S}_{24} = 24$ . This model was analyzed extensively in [15, 16].

- A  $d$ -dimensional model, where the breaking of the right-moving supersymmetries occurs via the coupling to the right-moving space-time fermion number  $F_R$  of the momentum and winding charges associated to a single factorized cycle, whose radius we denote by  $R_9$  [13]:

$$\begin{aligned} \frac{1}{(\eta\bar{\eta})^8} \frac{1}{2} \sum_{\bar{a}, \bar{b}} (-1)^{\bar{a}+\bar{b}} \frac{\bar{\theta}[\frac{\bar{a}}{\bar{b}}]^4}{\bar{\eta}^4} \Gamma_{(10-d, 10-d)[\frac{\bar{a}}{\bar{b}}]} &= \frac{\Gamma_{(9-d, 9-d)}}{(\eta\bar{\eta})^8} \\ &\times \sum_{m_9, n_9} \left( \bar{V}_8 \Gamma_{m_9, 2n_9}(R_9) + \bar{O}_8 \Gamma_{m_9+\frac{1}{2}, 2n_9+1}(R_9) - \bar{S}_8 \Gamma_{m_9+\frac{1}{2}, 2n_9}(R_9) - \bar{C}_8 \Gamma_{m_9, 2n_9+1}(R_9) \right). \end{aligned} \quad (2.7)$$

The breaking of the right-moving supersymmetries has a similar algebraic structure to the temperature breaking that acts on the left-moving characters, see Eq. (2.1). The Euclidean partition function is invariant under the  $R_9 \leftrightarrow R_0$  exchange [13]. The fermionic point



$R_9 = R_c$  corresponds to an extended symmetry point, with the  $U(1)_R$  gauge symmetry associated with the  $X^9$ -cycle getting enhanced to  $[SU(2)_R]_{k=2}$ . As we already stated, finite temperature and quantum effects give rise to an effective potential that stabilizes dynamically the value of the  $R_9$  modulus at the extended symmetry (fermionic) point [22, 23]. This behavior is *drastically different than that of the thermal  $R_0$  modulus*, where the back-reaction in the Lorentzian drives a cosmological evolution towards smaller temperatures. All other spectator moduli are either stabilized at extended symmetry points or frozen at values of order unity [23, 24]. The string coupling is taken to be sufficiently weak. As we will show, it remains smaller than a critical value during the induced cosmological evolution. Despite the lack of *MSDS* structure in the higher dimensional cases, the massive states are characterized by asymptotic supersymmetry, thanks to the asymmetric nature of the left- and right-moving supersymmetry breakings. This fact also explains the absence of tachyons from the spectrum of thermal masses [13, 17].

In the two dimensional Hybrid model, the genus-1 partition function takes a very simple form, which makes transparent the conical structure of the thermal partition function. The expression can be derived beyond any  $\alpha'$  approximation, thanks to the unbroken *MSDS* symmetry characterizing the right-moving sector, and it is given by [15, 16]:

$$\frac{Z_{\text{Hybrid}}}{V_1} = 24 \times \left( R_0 + \frac{1}{2R_0} \right) - 24 \times \left| R_0 - \frac{1}{2R_0} \right| = 24\sqrt{2} e^{-|\sigma|}. \quad (2.8)$$

The essential feature is a discontinuity in the first derivative of  $Z_{\text{Hybrid}}$  as a function of the thermal modulus  $\sigma$ , signaling a phase transition between the two dual “momentum” and “winding” regimes. The discontinuity occurs at the self-dual, fermionic point  $\sigma = 0$ , and it is sourced by the 24 complex lowest mass states in the  $O_8\bar{V}_{24}$ -sector, which become massless precisely at this point. Due to the unbroken *MSDS* right-moving structure, there are exact cancellations between the massive fermionic and massive bosonic oscillator states, and so both regimes at  $|\sigma| > 0$  comprise phases where the equation of state is effectively that of massless thermal radiation in two dimensions. Despite the cancellations in the massive sector, stringy behavior survives, with states carrying both momentum and winding charges becoming massless, inducing a phase transition at the critical point  $\sigma = 0$ .

In terms of the duality invariant temperature, the partition function is simply given by

$$\frac{Z_{\text{Hybrid}}}{V_1} = 48\pi T, \quad (2.9)$$

attaining a maximal value at the critical temperature  $T_c$ . The existence of this maximal temperature is the crucial difference from field theory thermal models where the temperature is unbounded. The numerical coefficient in Eq. (2.9) is easily understood as follows. In  $d$  dimensions, the partition function corresponding to massless thermal radiation can be written as

$$\frac{Z}{V_{d-1}} = n^* \Sigma_d T^{d-1}, \quad (2.10)$$

where  $n^*$  is given in terms of the numbers of the initially massless bosons and fermions and  $\Sigma_d$  is the Stefan-Boltzmann constant:

$$n^* = n^B + n^F \frac{2^{d-1} - 1}{2^{d-1}}, \quad \Sigma_d = \frac{\Gamma(d/2)}{\pi^{d/2}} \zeta(d). \quad (2.11)$$

Taking into account that initially there are  $8 \times 24$  massless bosons and  $8 \times 24$  massless fermions in the two dimensional Hybrid model and that  $\Sigma_2 = \pi/6$ , we obtain  $n_H^* \Sigma_2 = 48\pi$ .

We now proceed to analyze the  $d$ -dimensional model. The non-analytic, conical structure of the partition function occurs at the fermionic point  $R_0 = R_c$ . We can identify it by computing  $Z$  for  $R_0 > R_c$  and also for  $R_0 < R_c$ , and then utilize thermal duality to connect the two regimes so as to obtain an expression valid for all radii (see Refs [15, 16] for detailed discussions concerning the Hybrid model and also [9] for the two-dimensional Heterotic strings). For  $R_0 > R_c$ , we Poisson resum over the momentum quantum number  $m_0$ , and map the integral over the fundamental domain to an integral over the strip [25] involving the  $(\tilde{m}_0, n_0) = (2\tilde{k} + 1, 0)$  orbits only. In the strip representation, the winding contributions and in particular those of the  $O_8$  and  $C_8$  sectors to the fundamental domain integral, are mapped to  $V_8$ - and  $S_8$ -sector momentum contributions in the UV region of the strip,  $\tau_2 < 1$ . Mapping the integral over the fundamental domain to an integral over the strip as in [15, 16] (see also [25]), we obtain

$$\begin{aligned} \frac{Z}{V_{d-1}} = & 2R_0 \sum_{\tilde{k}=0}^{\infty} \int_{||} \frac{d^2\tau}{4(2\pi)^{d-1}\tau_2^{1+d/2}} e^{-(2\tilde{k}+1)^2 \frac{\pi R_0^2}{\tau_2}} \frac{\theta_2^4}{\eta^{12}} \\ & \times \frac{\Gamma(9-d, 9-d)}{\bar{\eta}^8} \sum_{m_9, n_9} \left( \bar{V}_8 \Gamma_{m_9, 2n_9} + \bar{O}_8 \Gamma_{m_9 + \frac{1}{2}, 2n_9 + 1} - \bar{S}_8 \Gamma_{m_9 + \frac{1}{2}, 2n_9} - \bar{C}_8 \Gamma_{m_9, 2n_9 + 1} \right) \Big|_{R_9=R_c}, \end{aligned} \quad (2.12)$$

where the moduli associated with the  $\Gamma_{(9-d, 9-d)}$  lattice are taken to be of order unity.

For large  $R_0$ , we split the contributions to the integral in two pieces: (i) the contribution

of the thermally excited massless bosons and fermions and (ii) the contributions of the massive states.

(i) *Massless contributions:*

The contribution of the initially massless bosons and fermions is given by

$$\begin{aligned} I_{\text{massless}} &= 2R_0 \sum_{\tilde{k}=0}^{\infty} \int_0^{\infty} \frac{d\tau_2}{2(2\pi)^{d-1} \tau_2^{1+d/2}} e^{-(2\tilde{k}+1)^2 \frac{\pi R_0^2}{\tau_2}} 16 \times (8+2), \\ &= 8 \times (8+2) \frac{2^d - 1}{2^{d-1}} \frac{\Gamma(d/2)}{\pi^{d/2}} \frac{\zeta(d)}{\beta^{d-1}} = n^* \Sigma_d \frac{1}{\beta^{d-1}}. \end{aligned} \quad (2.13)$$

(ii) *Massive contributions:*

To determine the contribution of the massive states, we must first compute the integral over  $\tau_1$ , which imposes level matching. The integral over  $\tau_2$  gives the final result in terms of Bessel functions. Due to the presence of the gravito-magnetic fluxes, there are alternating signs between the contributions of states in the  $F_R$ -even and  $F_R$ -odd sectors (right-moving bosons and right-moving fermions). The massive contributions become

$$I_{\text{massive}} = \frac{2}{(2\pi)^{d/2} \beta^{d/2-1}} \sum_i \sum_{\tilde{k}=0}^{\infty} (-1)^{F_R} \frac{|m_i|^{d/2}}{(2\tilde{k}+1)^{d/2}} K_{d/2} \left( (2\tilde{k}+1) \beta m_i \right), \quad (2.14)$$

where the first sum is over individual degenerate boson/fermion pairs, having the same right-moving fermion number. Thanks to the  $(-1)^{F_R}$  alternating signs, the effective density of states gets reduced drastically, as compared to the canonical thermal ensemble. This is a signal of right-moving asymptotic supersymmetry, replacing the exact right-moving *MSDS* structure of the Hybrid models. The massless sector on the other hand contributes identically as in the canonical thermal ensemble. For  $\beta \gg \beta_c$ , the arguments of the Bessel functions are large, leading to an exponential suppression of the contributions of all massive states. In this regime, we recover the characteristic behavior of massless thermal radiation in  $d$  dimensions:

$$\frac{Z}{V_{d-1}} \sim I_{\text{massless}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left( \frac{\beta_c}{\beta} \right)^{d-1}. \quad (2.15)$$

By thermal duality, the behavior for  $\beta \ll \beta_c$  is dual-to-thermal, yielding

$$\frac{Z}{V_{d-1}} \sim \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left( \frac{\beta}{\beta_c} \right)^{d-1}. \quad (2.16)$$

This T-dual result for  $R_0 \ll R_c$  can also be obtained if we first Poisson resum over the winding number  $n_0$  and utilize the momentum-unfolding to map the integral over the fundamental domain to an integral over the strip.

To complete the discussion, we have to analyze the behavior of the partition function in the intermediate regime when  $R_0$  is close but still larger than the fermionic point  $R_c$ . In this case, we must examine the contribution of the  $\tau_2 \rightarrow 0$  region to the integral (2.12), due to the exponential growth in the density of massive states in each sector of definite  $F_R$  parity separately. The individual contributions of the massive states are larger in this region. However, as we will see the contribution from this region is drastically reduced due to right-moving asymptotic supersymmetry, whose origin is the insertion of the  $(-1)^{F_R}$  phase. This property also explains the absence of physical tachyons from the spectrum of thermal masses. The asymptotic  $\tau_2 \rightarrow \infty$  region is dominated by the lightest string states giving rise to the thermal massless radiation contribution. As we show below this contribution will turn out to be the dominant one.

To proceed further we need to determine the  $\tau \rightarrow 0$  limit of the integrand in Eq. (2.12). To this end, it is convenient to rewrite the integrand in terms of shifted lattices  $\Gamma_g^{[h]}$ :

$$\begin{aligned} \frac{Z}{V_{d-1}} = & -2R_0 \sum_{\tilde{k}=0}^{\infty} \int_{||} \frac{d^2\tau}{4(2\pi)^{d-1}\tau_2^{1+d/2}} e^{-(2\tilde{k}+1)^2 \frac{\pi R_0^2}{\tau_2}} \frac{\theta_2^4}{\eta^{12}} \\ & \times \frac{\Gamma_{(9-d,9-d)}}{\bar{\eta}^8} \frac{1}{\bar{\eta}^4} \left( \Gamma_{[1]}^{[1]} \bar{\theta}_3^4 - \Gamma_{[0]}^{[1]} \bar{\theta}_4^4 - \Gamma_{[1]}^{[0]} \bar{\theta}_2^4 \right) \Big|_{R_9=R_c}, \end{aligned} \quad (2.17)$$

where

$$\Gamma_{[\tilde{g}]}^{[h]}(R) = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde{m},n} e^{-\frac{\pi R^2}{\tau_2} |(2\tilde{m}+\tilde{g})+(2n+h)\tau|^2}. \quad (2.18)$$

We then apply the modular transformation  $\tau \rightarrow \tilde{\tau} = -1/\tau$  to the following expression

$$\begin{aligned} \frac{1}{\tau_2^{d/2-1}} \frac{\theta_2^4}{\eta^4} \frac{\Gamma_{(9-d,9-d)}}{(\eta\bar{\eta})^8} \frac{1}{\bar{\eta}^4} \left( \Gamma_{[1]}^{[1]} \bar{\theta}_3^4 - \Gamma_{[0]}^{[1]} \bar{\theta}_4^4 - \Gamma_{[1]}^{[0]} \bar{\theta}_2^4 \right) = \\ \frac{1}{\tilde{\tau}_2^{d/2-1}} \left[ \frac{\theta_4^4}{\eta^4} \frac{\Gamma_{(9-d,9-d)}}{(\eta\bar{\eta})^8} \frac{1}{\bar{\eta}^4} \left( \Gamma_{[1]}^{[1]} \bar{\theta}_3^4 - \Gamma_{[0]}^{[1]} \bar{\theta}_4^4 - \Gamma_{[1]}^{[0]} \bar{\theta}_2^4 \right) \right](\tilde{\tau}), \end{aligned} \quad (2.19)$$

which appears in the integrand of Eq. (2.17). The last expression can be expanded in powers of  $\tilde{q} = e^{2\pi\tilde{\tau}}$ , in the limit  $\tilde{q} \rightarrow 0$ . For  $R_9 = R_c$ , we obtain the following leading terms

$$-\frac{1}{\tilde{\tau}_2^{d/2-1}} \left( \tilde{q}^{-\frac{1}{2}} - 8 \right) \left( 8 + 2 + 2 \tilde{q}^{\frac{1}{2}} \tilde{q}^{-\frac{1}{2}} \right). \quad (2.20)$$

Essentially, only the left/right level matched terms contribute due to the integration over  $\tau_1$ . Keeping these only, we get

$$\frac{1}{\tilde{\tau}_2^{d/2-1}} 8 \times (8 + 2) = \tau_2^{d/2-1} 8 \times (8 + 2). \quad (2.21)$$

The absence of exponential growth in this factor is the signal of asymptotic supersymmetry, as was already stated before. Notice that for  $d > 2$ , this factor goes to zero as a power law. So, the contribution from this region is estimated to be

$$-8 \times (8 + 2) \frac{R_c}{(2\pi)^{d-1}} \int_{t^*}^{\infty} \frac{d\tilde{\tau}_2}{\tilde{\tau}_2^{1+d/2}} e^{-\pi R_c^2 \tilde{\tau}_2} \sim \mathcal{O}\left(\frac{e^{-\pi L}}{L^{\frac{d+2}{2}}}\right), \quad (2.22)$$

where  $t^* = \frac{L}{R_c^2}$  is a sufficiently large cutoff,  $L \gg 1$ . The overall contribution is thus exponentially suppressed.

Therefore the behavior of  $Z$  for  $R_0 > R_c$  is controlled by the thermally excited initially massless states everywhere and up to the critical point. The same conclusion can be reached in the regime  $R_0 < R_c$  by thermal duality. Gluing the two regimes in a duality invariant way gives

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} e^{-(d-1)|\sigma|} = n^* \Sigma_d T^{d-1}, \quad (2.23)$$

modulo the exponentially suppressed contributions in the three effective field theory regimes. This result also implies that in each of the two thermal phases the various thermodynamical quantities enjoy the standard monotonicity properties as functions of the temperature, with the specific heat being positive up to the critical point.

To conclude, the above result reveals a universal conical structure at  $\sigma = 0$ , irrespectively of the dimensionality of the model. All of the above manipulations, including thermal duality, amount to approximating the last line of equation (2.12) with a factor of order unity:

$$\frac{Z}{V_{d-1}} = 2R_c e^{|\sigma|} \sum_{\tilde{k}=0}^{\infty} \int_{||} \frac{d^2 \tau}{4(2\pi)^{d-1} \tau_2^{1+d/2}} \exp\left(\frac{-(2\tilde{k}+1)^2 \pi R_c^2}{\tau_2} e^{2|\sigma|}\right) \frac{\theta_2^4}{\eta^{12}} (8+2). \quad (2.24)$$

Thanks to the analytic properties of the left-moving characters, only the massless level contributes in the above integral. This is the generalization of the Hybrid model result [15,16] to arbitrary dimensions via right-moving asymptotic supersymmetry.

### 3 Effective action(s) up to genus-1

From the thermal effective field theory point of view, there are at least three different effective actions associated to three possible  $\alpha'$ -like expansions, each being valid in one of the three characteristic regimes. Namely:

$$\frac{R_0}{R_c} \gg 1 \quad (\sigma \gg 0), \quad \frac{R_c}{R_0} \gg 1 \quad (-\sigma \gg 0), \quad \left| \frac{R_0}{R_c} - \frac{R_c}{R_0} \right| \ll 1 \quad (\sigma \simeq 0). \quad (3.1)$$

Taking into account the behavior of the thermal partition function in each regime, the thermal duality symmetry, as well as the branes which glue the dual “momentum” and “winding” phases, we will construct a  $d$ -dimensional effective cosmological action valid in all regimes simultaneously and derive the associated equations of motion. In the Lorentzian, the branes are spacelike, appearing at any time when the temperature reaches its critical maximal value  $T_c$ . As we will argue, these source localized negative pressure contributions to the effective action.

In the two asymptotic regimes ( $\sigma \rightarrow \pm\infty$ ) dominated by the light thermal momenta and the light thermal windings respectively, the effective action admits the well known sigma-model descriptions, defined via the corresponding  $\alpha'$ -expansions. Thanks to thermal duality, both asymptotic regimes can be simultaneously described by *a unique expansion in terms of the duality invariant temperature*  $T = T_c e^{-|\sigma|}$ . All thermodynamical quantities such as the temperature, the energy density and the pressure are given in terms of manifestly duality invariant expressions involving the absolute value of the thermal modulus  $|\sigma|$ . In the Euclidean, the regime  $|\sigma| \rightarrow 0$  is well described in terms of the  $[SU(2)_L]_{k=2}$  CFT associated with the fermionic extended symmetry point. In this regime, we have to include the contributions of the extra massless thermal states, responsible for the phase transition, both at the genus-0 and genus-1 levels. The genus-0 contributions admit a brane interpretation with a tension determined by the allowable non-trivial backgrounds of the extra massless thermal scalars,  $\partial_\mu \varphi^I \neq 0$ , where the gradients are along the directions transverse to Euclidean time.

During the cosmological evolution, the thermal modulus  $\sigma$  acquires non-trivial time-dependence,  $\sigma(\tau)$ . Since all fields are functions of  $|\sigma(\tau)|$ , their second time-derivatives may give rise to localized singular terms proportional to

$$\delta(\sigma(\tau)) = \sum_i \frac{d\tau}{d\sigma} \delta(\tau - \tau_i), \quad (3.2)$$

where the temperature reaches its critical value at times  $\tau_i$  ( $i = 1, \dots, n$ ) so that  $\sigma(\tau_i) = 0$ . As we will see, these singularities are naturally resolved by the presence of the spacelike branes, localized at the times  $\tau_i$ .

The relevant representations of the winding-like field theory are space-time left-moving Vectors  $V_8$  and space-time anti-Spinors  $C_8$ . On the other hand, in the momentum-like field theory, the relevant operators are the left-moving Vectors  $V_8$  and space-time Spinors  $S_8$ .

At the branes, the theory is self-dual and both the Spinor and anti-Spinor representations coexist together with extra massless states with non-trivial momentum and winding charges,  $(p_L, p_R) = (\pm 1, 0)$ , triggering the transition of the winding-like field theory based on  $V_8 - C_8$  to the momentum-like one based on  $V_8 - S_8$ .

The above ingredients lead to an effective  $d$ -dimensional dilaton-gravity action (up to the genus-1 level), able to describe simultaneously and in a consistent way the three regimes:

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 + \mathcal{S}_{\text{brane}}, \quad (3.3)$$

where

$$\begin{aligned} \mathcal{S}_0 &= \int d^d x e^{-2\phi} \sqrt{-g} \left( \frac{1}{2} \mathcal{R} + 2(\nabla\phi)^2 \right), \\ \mathcal{S}_1 &= \int d^d x \sqrt{-g} P, \\ \mathcal{S}_{\text{brane}} &= - \sum_i \int d^d x \sqrt{g_\perp} e^{-2\phi} \kappa_i \delta(\tau - \tau_i). \end{aligned} \quad (3.4)$$

- $\mathcal{S}_0$  is the genus-0 dilaton-gravity action written in the string frame.
- $\mathcal{S}_1$  is the genus-1 contribution of the thermal effective potential  $-P$ .
- $\mathcal{S}_{\text{brane}}$  is the spacelike brane contribution at the phase transition giving rise to localized negative pressure. It is sourced by the additional massless scalars  $\varphi^I$  from the  $O_8\bar{V}_8$  and  $O_8\bar{O}_8$  sectors at the extended symmetry point  $\sigma = 0$ . These extra massless states parametrize a manifold, which up to discrete identifications, takes the form of a coset space,

$$\mathcal{M}^{2q}(\varphi^I) = \frac{SO(2, q)}{SO(2) \times SO(q)}, \quad (3.5)$$

of real dimension  $2q$  which depends on the number of massless states coming from the right-moving (non-supersymmetric) sector. In a suitable parametrization, the associated Kähler potential is given by

$$K = -\ln \left( (Y^0 + \bar{Y}^0)^2 - (Y^\alpha + \bar{Y}^\alpha)^2 \right), \quad \alpha = 1, \dots, q-1, \quad (3.6)$$

with the metric given in terms of the holomorphic and anti-holomorphic derivatives of  $K$ :

$$ds^2(\mathcal{M}^{2q}) = \partial_\alpha \partial_{\bar{\beta}} K dY^\alpha dY^{\bar{\beta}} = K_{\alpha\bar{\beta}} dY^\alpha dY^{\bar{\beta}}.$$

The smallest possible value for  $q$  is  $10 - d$ , which occurs when there is no extended right-moving gauge symmetry. In the models of [13] described in detail in section 2,  $q = 2 + (10 - d)$ ,

with the gauge symmetry of the right-moving sector being extended to  $\mathcal{H}_R = SU(2) \times U(1)^{9-d}$  of dimension  $3 + (9 - d) = q$ . The maximum value for  $q$  is  $3(10 - d)$ , occurring when the gauge symmetry of the right-moving sector is extended to  $\mathcal{H}_R$  of dimension  $3(10 - d)$ . In the simplest models with this property,  $\mathcal{H}_R = SU(2)^{10-d}$ .

### 3.1 The S-brane action

The microscopic origin of the brane term in Eq. (3.3) follows from the underlying description of the system at the extended symmetry point. We will be interested in obtaining homogeneous and isotropic solutions of the bulk. The scalars  $\varphi^I$  give rise to a tree-level localized action

$$\mathcal{S}_{\text{brane}} = - \int d\sigma d^{d-1}x \sqrt{g_{\perp}} e^{-2\phi} g^{\hat{\mu}\hat{\nu}} G_{IJ} \partial_{\hat{\mu}} \varphi^I \partial_{\hat{\nu}} \varphi^J \delta(\sigma), \quad (3.7)$$

where  $\hat{\mu} = 1, \dots, d-1$ ,  $g_{\perp} = \det g_{\hat{\mu}\hat{\nu}}$  and  $G_{IJ}$  (or  $K_{\alpha\bar{\beta}}$ ) is the metric in the field configuration space  $\mathcal{M}^{2q}$ . The equations of motion of the scalars  $\varphi^I$  take the form

$$2\partial_{\hat{\mu}}(e^{-2\phi} \sqrt{g_{\perp}} g^{\hat{\mu}\hat{\nu}} G_{IJ} \partial_{\hat{\nu}} \varphi^J) - e^{-2\phi} \sqrt{g_{\perp}} g^{\hat{\mu}\hat{\nu}} (\partial_I G_{KJ}) \partial_{\hat{\mu}} \varphi^K \partial_{\hat{\nu}} \varphi^J = 0. \quad (3.8)$$

Our aim is to establish that these equations admit non-trivial solutions which are consistent with the homogeneity and isotropy requirements and yield the localized brane contributions in the effective action as described by Eqs (3.3) and (3.4). To this end, it suffices that at each instant  $\tau_i$  ( $i = 1, \dots, n$ ), when the temperature reaches its critical value, the induced metric be proportional to the spatial metric:

$$h_{\hat{\mu}\hat{\nu}} \equiv G_{IJ} \partial_{\hat{\mu}} \varphi^I \partial_{\hat{\nu}} \varphi^J = \frac{\kappa_i}{d-1} g_{\hat{\mu}\hat{\nu}}, \quad (3.9)$$

where the  $\kappa_i$ 's are positive constants. When this happens, the stress tensor of the scalars is consistent with the symmetries of the spatial metric, and therefore with homogeneity and isotropy. Moreover, the action (3.7) takes the familiar form of the Nambu-Goto action for branes,

$$\begin{aligned} \mathcal{S}_{\text{brane}} &= - \sum_i \kappa_i \int d^d x e^{-2\phi} \sqrt{g_{\perp}} \delta(\tau - \tau_i) \\ &= - \sum_i (d-1)^{\frac{d-1}{2}} \kappa_i^{\frac{3-d}{2}} \int d^d x e^{-2\phi} \sqrt{\det(G_{IJ} \partial_{\hat{\mu}} \phi^I \partial_{\hat{\nu}} \phi^J)} \delta(\tau - \tau_i), \end{aligned} \quad (3.10)$$

where we have used Eq. (3.2). Thus,  $\kappa_i$  is interpreted as a brane tension.



To exhibit the solutions, we first consider the following embedding of space  $\Omega^{d-1}(x^{\hat{\mu}})$  into the field configuration space,  $\Omega^{d-1}(x^{\hat{\mu}}) \rightarrow \mathcal{M}^{2q}(\varphi^I)$  :

$$\partial_{\hat{\mu}}\varphi^I = \delta_{\hat{\mu}}^I, \quad \hat{\mu}, I = 1, \dots, d-1, \quad \text{and} \quad \varphi^I = \text{const.}, \quad I = d, \dots, 2q. \quad (3.11)$$

The embedding exists provided that the dimensionality of the scalar field manifold is bigger or equal to the dimension of space:  $2q \geq d-1$ . It also implies that the induced metric satisfies:  $h_{\hat{\mu}\hat{\nu}} = G_{\hat{\mu}\hat{\nu}}$ . Thus Eq. (3.9) imposes that the spatial metric  $g_{\hat{\mu}\hat{\nu}}$  and  $G_{\hat{\mu}\hat{\nu}}$  are isomorphic

$$G_{\hat{\mu}\hat{\nu}} = \frac{\kappa_i}{d-1} g_{\hat{\mu}\hat{\nu}}, \quad (3.12)$$

upon the identification  $\varphi^{\hat{\mu}} = x^{\hat{\mu}}$ . Under the above circumstances, the field equations of motion (3.8) for  $I = \hat{\mu} \leq d-1$  become:

$$2\partial_{\hat{\mu}}(e^{-2\phi}\sqrt{g_{\perp}}) - e^{-2\phi}\sqrt{g_{\perp}} g^{\hat{\sigma}\hat{\nu}}\partial_{\hat{\mu}}g_{\hat{\nu}\hat{\sigma}} = 0 \quad \implies \quad \partial_{\hat{\mu}}\phi = 0, \quad (3.13)$$

consistently with the homogeneity of the dilaton field  $\phi$ , and where we have used the identity

$$g^{\hat{\sigma}\hat{\nu}}\partial_{\hat{\mu}}g_{\hat{\nu}\hat{\sigma}} = 2\partial_{\hat{\mu}}\log\sqrt{g_{\perp}}.$$

Eqs (3.8) for  $I > d-1$  must also be satisfied. The above discussion makes it clear that the geometrical structure of the field manifold  $\mathcal{M}^{2q}(\varphi^I)$  is crucial, since it constrains the possible embeddings of  $\Omega^{d-1}(x^{\hat{\mu}})$  into  $\mathcal{M}^{2q}(\varphi^I)$ . We are mainly interested for the isotropic embeddings of the hyperbolic space  $H^{d-1}$  and the flat space  $F^{d-1}$ , with curvature and metric given by

$$(i) \ H^{d-1} : k = -1 \ (\text{for } d > 2), \quad g_{\hat{\mu}\hat{\nu}} = \frac{a(\tau_i)^2}{(x^{d-1})^2} \delta_{\hat{\mu}\hat{\nu}}, \quad (ii) \ F^{d-1} : k = 0, \quad g_{\hat{\mu}\hat{\nu}} = a(\tau_i)^2 \delta_{\hat{\mu}\hat{\nu}}, \quad (3.14)$$

where  $a(\tau_i)$  is the scale factor at time  $\tau_i$ .

(i) *Hyperbolic embedding*  $H^{d-1} \rightarrow \mathcal{M}^{2q}$

The hyperbolic embedding turns out to be naturally realized thanks to the geometrical structure of the field manifold  $\mathcal{M}^{2q}$ . Indeed it is sufficient to utilize the sub-manifold  $K^q(y^I) \subset \mathcal{M}^{2q}(Y^I)$  with  $y^I = \text{Re } Y^I$  ( $I = 0, \dots, q-1$ ) non trivial and  $\omega^I = \text{Im } Y^I$  fixed. The sub-manifold  $K^q(y^I)$  naturally contains the desired  $H^{q-1}$  factor:

$$K^q \equiv H^{q-1} \times SO(1,1) = \frac{SO(1, q-1)}{SO(q-1)} \times SO(1,1). \quad (3.15)$$

The metric on  $K^q$  follows from the Kähler metric  $K_{\alpha\bar{\beta}}$  and takes the form:

$$ds^2(K^q) = \frac{-(dy^0)^2 + (dy^I)^2}{2r^2} + \frac{(dr)^2}{r^2} \quad \text{with} \quad r^2 \equiv (y^0)^2 - (y^I)^2. \quad (3.16)$$

The imaginary parts  $\omega^I$  are frozen consistently with all equations of motion. The metric of the  $(q-1)$ -dimensional hyperboloid  $H^{q-1}$  is obtained by writing

$$-(dy^0)^2 + (dy^I)^2 = -dr^2 + r^2 (dH^{q-1})^2, \quad (3.17)$$

which shows the explicit factorization of  $K^q$ :

$$ds^2(K^q) = \frac{1}{2} (d\zeta^2 + (dH^{q-1})^2), \quad \zeta = \ln r. \quad (3.18)$$

Fixing further the field  $\zeta$  to be constant, as allowed by the equations of motion, the embedding of the spatial hyperboloid  $H^{d-1}(x^{\hat{\mu}})$  into  $H^{q-1}(\varphi^i)$  is automatic. This can be done, provided that  $q \geq d$ ,  $\varphi^{\hat{\mu}} = x^{\hat{\mu}}$  and the extra  $q-d$  fields  $\varphi^i, i = d, \dots, q-1$  are also frozen as allowed by the equations of motion:

$$ds^2(H^{q-1}) = \frac{(d\varphi^{\hat{\mu}})^2}{(\varphi^{d-1})^2} + \frac{(d\varphi^i)^2}{(\varphi^{d-1})^2} \quad \implies \quad ds^2(H^{d-1}) = \frac{(d\varphi^{\hat{\mu}})^2}{(\varphi^{d-1})^2}. \quad (3.19)$$

When  $q = 2 + (10-d)$  as in the models of [13], the constraint  $d \leq q$  implies  $d \leq 6$ , while when  $q$  takes the maximal value  $3(10-d)$ , we must have  $d \leq 7$ . These embeddings satisfy the relation (3.9) with the tension  $\kappa_i$  fixed in terms of the scale factor:  $\kappa_i = (d-1)/2a(\tau_i)^2$ . Such a tension leads to highly curved cosmological solutions with Ricci curvature of order one, as the value of  $a(\tau_i)$  is fixed to be of order one by the cosmological equations (see section 4.2).

To relax the constraint on the tension  $\kappa_i$ , we take into account discrete identifications in the field configuration space in order to obtain solutions with a non-trivial wrapping number. With suitable discrete identifications, the submanifold parametrized by the fields  $\varphi^I$  ( $I = 1, \dots, d-1$ ) becomes a finite volume hyperbolic space of the form  $H^{d-1}/\Gamma$ , where  $\Gamma$  is a subgroup of the discrete duality group  $SO(2, q; \mathbb{Z})$  of  $\mathcal{M}^{2q}$ . The two dimensional examples correspond to the familiar higher genus Riemann surfaces. Homogeneous compact hyperbolic manifolds in dimension  $\geq 3$  are characterized by the property of rigidity, which implies that there are no massless shape moduli [26], and are locally isotropic. The volume is determined by the radius of curvature  $L$  and the topology of the manifold:  $\text{Vol}(H^{d-1}/\Gamma) = L^{d-1}e^\alpha$ , where

$\alpha$  is a constant determined by the topology and  $L \sim 1$  for the metric on the field subspace. The topological factor  $e^\alpha$  is unbounded from above. Taking the spatial manifold  $\Omega^{d-1}(x^{\hat{\mu}})$  to also be a compact hyperbolic manifold of large volume (and suitable topology <sup>1</sup>), allows for embeddings with arbitrary wrapping number  $w$ . Consequently the brane action is finite, given by

$$S_{\text{brane}}^i = -\frac{c^* w}{a(\tau_i)^2} e^{-2\phi_i} \text{Vol}(\Omega^{d-1}) = -\kappa_i \int dx^{d-1} e^{-2\phi_i} \sqrt{g_{\perp}}, \quad (3.20)$$

implying that the brane tension is  $\kappa_i = c^* w^2 / a(\tau_i)^2$ . Here  $c^*$  is a factor determined by the topology. The wrapping number  $w$  being arbitrary, the tension can be kept arbitrary.

(ii) *Flat embedding*  $F^{d-1} \rightarrow \mathcal{M}^{2q}$

A way to realize the flat embedding is by utilizing a  $(d-1)$ -dimensional flat section  $F^{d-1}(\varphi^{\hat{\mu}})$  of  $\mathcal{M}^{2q}$ . The isotropic embedding of flat space  $F^{d-1}(x^{\hat{\mu}})$  into the flat section  $F^{d-1}(\varphi^{\hat{\mu}})$  is defined by

$$\partial_{\hat{\nu}} \varphi^{\hat{\mu}} = \sqrt{\frac{2\kappa_i}{d-1}} a(\tau_i) \delta_{\hat{\nu}}^{\hat{\mu}}, \quad (3.21)$$

giving rise to Eq. (3.9) and the brane action with tension  $\kappa_i$ . We display below examples of such flat sections.

- For  $d = 2$ , we can utilize the  $SO(1,1)_{\zeta}$  factor of the submanifold  $K^q$  parametrized by the field  $\zeta$  in Eq. (3.18) to carry out the embedding  $F^1 \rightarrow \mathcal{M}^{2q}$ . All other fields are frozen consistently with the equations of motion.
- For  $d = 3$ , we utilize the  $SO(1,1)_{\zeta} \times SO(1,1)_{\xi}$  submanifold of  $K^q$  parametrized by the fields  $\zeta$  and  $\xi \equiv \ln \varphi^{d-1}$ , see Eq. (3.19).
- For  $d \geq 4$  we utilize the flat section of  $H^{d-1}(u^i)$  obtained in the limit  $u^{d-1} \rightarrow \infty$ , together with the  $SO(1,1)_{\zeta}$  factor. To this end, we define the rescaled fields  $\varphi^i$  by setting  $u^i \equiv u^{d-1} \varphi^i$  ( $i = 1, \dots, d-2$ ), in order to obtain:

$$\begin{aligned} \frac{1}{2} [d\zeta^2 + (dH^{d-1})^2] &= \frac{1}{2} [d\zeta^2 + (d\varphi^i)^2] + \frac{1}{u^{d-1}} \varphi^i d\varphi^i du^{d-1} + \frac{1 + (\varphi^i)^2}{2(u^{d-1})^2} (du^{d-1})^2 \\ &= \frac{1}{2} [d\zeta^2 + (d\varphi^i)^2] + \mathcal{O}\left(\frac{1}{u^{d-1}}\right). \end{aligned} \quad (3.22)$$

The field  $\zeta$  and the  $d-2$  fields  $\varphi^i$  are utilized to realize the flat embedding of Eq. (3.21). The  $2q - (d-1)$  extra fields are frozen, consistently with the equations of motion (3.8),

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<sup>1</sup>See e.g. [27, 28] for discussions concerning this possibility in the context of cosmological and other applications.

including that of  $u^{d-1}$  in the limit  $u^{d-1} \rightarrow \infty$ . The realization of this isotropic embedding imposes the constraint  $d \leq q$ . When  $q = 2 + (10 - d)$  as in the models of [13], this implies  $d \leq 6$ , while when  $q$  takes the maximal value  $3(10 - d)$ ,  $d \leq 7$  is required.

Finally let us note that isotropic embeddings are also possible when space is isomorphic to the  $(d-1)$ -dimensional sphere  $S^{d-1}$ . These embeddings, the restrictions on the dimensionality of spacetime and the structure of the bulk solutions are currently under investigation. This completes our discussion about the origin of the brane contributions in the effective action.

### 3.2 Equations of motion

Looking for homogeneous and isotropic cosmological solutions in dimensions  $d \geq 2$ , the dilaton field is a function of time only and the metric

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 (d\Omega^{d-1})^2, \quad \phi = \phi(\tau) \quad (3.23)$$

involves the line element  $(d\Omega^{d-1})^2$  of the  $(d-1)$ -dimensional Einstein space with curvature  $k$ . To derive the equations of motion, we utilize the analytic expression for the Ricci scalar curvature

$$\mathcal{R} = \frac{2(d-1)}{N^2} \left[ \frac{\ddot{a}}{a} + \frac{(d-2)}{2} \left( H^2 + \frac{kN^2}{a^2} \right) - H \frac{\dot{N}}{N} \right] \quad \text{where} \quad H \equiv \frac{\dot{a}}{a}. \quad (3.24)$$

The pressure  $P$  is determined by the genus-1 Euclidean path integral  $Z$  (the thermal partition function) as

$$P = T_c e^{-|\sigma|} \frac{Z(|\sigma|)}{V_{d-1}}. \quad (3.25)$$

It is important to note that in the sigma-model frame,  $P$  is a function of the thermal modulus  $|\sigma|$  only and there is no dependence on the dilaton field  $\phi$ .

The above considerations lead us to the following equations of motion:

(i) *N-equation*

$$\frac{1}{2}(d-1)(d-2) \left( H^2 + k \frac{N^2}{a^2} \right) = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi} N^2 \rho, \quad (3.26)$$

where the energy density is given by<sup>2</sup>

$$\rho = -P - \frac{\partial P}{\partial |\sigma|} = -T_c e^{-|\sigma|} \frac{\partial}{\partial |\sigma|} \left( \frac{Z(|\sigma|)}{V_{d-1}} \right). \quad (3.27)$$

(ii) *a-equation*

$$(d-2)\frac{\ddot{a}}{a} + \frac{1}{2}(d-2)(d-3) \left( H^2 + k \frac{N^2}{a^2} \right) - (d-2)H \frac{\dot{N}}{N} =$$

$$2\ddot{\phi} + 2(d-2)H\dot{\phi} - 2\dot{\phi}^2 - 2\dot{\phi} \frac{\dot{N}}{N} - e^{2\phi} N^2 P + \sum_i \kappa_i N \delta(\tau - \tau_i). \quad (3.28)$$

(iii)  *$\phi$ -equation:*

$$\ddot{\phi} + (d-1)H\dot{\phi} - \dot{\phi}^2 - \dot{\phi} \frac{\dot{N}}{N} =$$

$$\frac{d-1}{2} \left( \frac{\ddot{a}}{a} + \frac{d-2}{2} \left( H^2 + k \frac{N^2}{a^2} \right) - H \frac{\dot{N}}{N} \right) - \frac{1}{2} \sum_i \kappa_i N \delta(\tau - \tau_i). \quad (3.29)$$

It is useful to disentangle the second derivatives of the scale factor and dilaton field in the last two equations. Of particular interest is the linear combination which leads

(ii)' *trace equation*

$$\dot{\phi}^2 - \frac{d-1}{2} \left( \frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) - \frac{1}{4}(d-1)(d-2) \left( H^2 + k \frac{N^2}{a^2} \right) = e^{2\phi} N^2 [\rho - (d-1)P] \quad (3.30)$$

and shows that the second derivative of the scale factor is always finite. In particular, this implies that  $\dot{a}$  is continuous, even at the branes localized at  $\tau = \tau_i$ . On the contrary, another equation for the dilaton is

(iii)'  *$\phi$ -equation modulo trace equation*

$$2\ddot{\phi} - 4\dot{\phi}^2 + 2(d-1)H\dot{\phi} - 2\dot{\phi} \frac{\dot{N}}{N} = e^{2\phi} N^2 [(d-1)P - \rho] - \sum_i \kappa_i N \delta(\tau - \tau_i), \quad (3.31)$$

which shows that the first time-derivative of the dilaton is discontinuous across  $\tau_i$ . This discontinuity is resolved by the presence of a spacelike brane, whose tension must satisfy

$$\kappa_i = 2 \frac{\dot{\phi}(\tau_{i-}) - \dot{\phi}(\tau_{i+})}{N(\tau_i)}. \quad (3.32)$$

---

<sup>2</sup>To derive  $\rho$ , we make use of  $\frac{\delta[N(\tau')P(|\sigma(\tau')|)]}{\delta N(\tau)} = \delta(\tau' - \tau)P + N \frac{\partial P}{\partial |\sigma|} \frac{\delta|\sigma(\tau')|}{\delta N(\tau)}$  where  $\frac{\delta|\sigma(\tau')|}{\delta N(\tau)} = \frac{\delta(\tau' - \tau)}{N}$ , as follows from the fact that  $\beta_c e^{|\sigma(\tau)|} \frac{dx^0}{d\tau} \equiv N(\tau)$ .

This resolution of the discontinuity via branes provides a novel mechanism in obtaining non-singular bouncing cosmologies, which in fact remain in a perturbative regime throughout the evolution. Such dynamical behavior is induced by the presence of spacelike branes providing localized negative contributions to the pressure

$$P_B = - \sum_i e^{-2\phi(\tau_i)} \kappa_i \delta(\tau_i), \quad (3.33)$$

thus evading the conditions set by the standard singularity theorems on realizing non-singular bouncing cosmologies [1].

Before solving the above cosmological equations, it is important to stress that these give rise to an integrable relation of fundamental interest, namely the Entropy conservation equation:

(s) *Entropy equation*

$$(\dot{\rho} + \dot{P}) + ((d-1)H + |\dot{\sigma}|)(\rho + P) = 0. \quad (3.34)$$

Integrating once, we obtain a quantity  $S$ , which we may interpret as the conserved thermal entropy in a comoving cell of volume  $a^{d-1}$ :

(s)' *Entropy conservation*

$$\frac{a^{d-1}}{T}(\rho + P) = S. \quad (3.35)$$

Since the energy density  $\rho$  and the pressure  $P$  are bounded, attaining their maximal values at the critical temperature  $T_c$ , the scale factor  $a$  is bounded from below, acquiring its minimal value at the critical point. Thus the big-bang singularity  $a = 0$  of general relativity is avoided in all such Hagedorn-free string models. The constant thermal entropy  $S$  can be computed in the asymptotic regime where the system is radiation dominated and the dilaton is constant:

$$\rho \sim (d-1)P \sim (d-1)n^* \Sigma_d T^d \quad \text{as} \quad |\sigma| \rightarrow \infty \quad \implies \quad S = d \gamma_\infty^{d-1} n^* \Sigma_d, \quad (3.36)$$

where the constant  $\gamma_\infty$  is given in terms of the asymptotic values of  $a$  and  $T$ ,  $\gamma_\infty = \lim_{|\sigma| \rightarrow \infty} aT$ .

As we demonstrated, the stringy thermal system is effectively radiation dominated in all regimes. As a result, an important consequence of the applicability of the entropy conservation in all regimes is that  $\gamma = aT$  is almost constant everywhere and up to the critical

temperature. The largeness of the entropy observed at late cosmological times implies that the size of the Universe at the critical point is already large. This fact guarantees the validity of our perturbative approach as we will see later, and also, the connection of the so called entropy and oldness problems of standard Big/Bang cosmology.

## 4 Stringy non-singular cosmologies

In this section, we exhibit non-singular cosmological solutions in various dimensions. We will show that the mechanism for the resolutions of both the Hagedorn and the initial singularity problems are generic in all space-time dimensions. This result follows from the universal properties of the partition function discussed in detail in sections 2 and 3. Namely:

- The thermal partition function has a conical singularity as a function of the thermal modulus  $\sigma$  at the critical point  $\sigma = 0$ , irrespectively of the space-time dimension. This implies a phase transition between the light-momentum and light winding-effective field theory regimes. At the phase transition, spacelike branes appear connecting the two asymptotic regimes.
- Across the branes, the behavior of the scale factor and dilaton field is such that  $\dot{a}$  is continuous, while there is a discontinuity in  $\dot{\phi}$  related to the brane tension, see Eq. (3.32).

Two-dimensional examples have already been exhibited in Ref. [16] in the framework of the Hybrid models, where the contributions of the massive modes to the partition function  $Z$  cancel exactly thanks to the *MSDS* structure characterizing the right-moving sector. In the higher dimensional models presented in section 2, the exact structure of the partition function is more involved. However, knowing the asymptotic behavior of  $Z$  as  $\sigma \rightarrow \pm\infty$ , as well as the asymptotic density of the right-moving states, allow us to determine the essential thermodynamical properties for all temperatures  $T \leq T_c$ . Indeed, thanks to the asymptotic right-moving supersymmetry characterizing the models, we can well-approximate  $Z$  with the contribution of the thermally excited massless states in both the  $\sigma > 0$  and the  $\sigma < 0$  regimes, as was explained in section 2. Thus, in what follows we set

$$\rho = (d-1)P = (d-1)n^*\Sigma_d T^d = (d-1)n^*\Sigma_d T_c^d e^{-d|\sigma|}. \quad (4.1)$$

This last expression captures the essential conical structure of the higher dimensional cases, generalizing the two dimensional Hybrid results. It allows us to obtain analytic cosmological solutions via spacelike branes, as in the two-dimensional case. The conical structure arises from matching the dual momentum and winding regimes, which admit distinct effective field theory descriptions. The intermediate  $\sigma = 0$  regime gives rise to the spacelike branes.

The brane interpretation gives us the possibility to look for branches of solutions defined in time intervals  $(\tau_i, \tau_{i+1})$ , with  $\sigma(\tau_i) = 0$ . These intervals are connected to each other by spacelike branes localized at  $\tau_i$ . From the world-sheet point of view the transition occurs via a condensation of massless thermal states carrying non-trivial momentum and winding charges associated to the vertex operators  $O_{\pm}$  defined in Eq. (2.4). The existence of these additional marginal operators at  $\sigma = 0$  gives rise to the transition between the thermal winding states at  $\sigma = 0_-$  and the thermal momentum states at  $\sigma = 0_+$  and vice-versa. The Universe may experience a series of such phase transitions. For a cosmological evolution to be consistent, the following constraints must be fulfilled:

1. The dilaton and scale factor must be continuous across the branes.
2. The scale factor must be bounded from below,  $a(\tau) \geq a_c$ , since  $T \propto 1/a$  cannot reach values larger than the maximal temperature  $T_c$ .
3. Across any brane,  $\dot{\phi}(\tau_{i-}) \geq \dot{\phi}(\tau_{i+})$  is required, since the discontinuities in the first time derivative of the dilaton are resolved by a positive brane tension (see Eq. (3.32)).
4. Since the first derivative of the scale factor is smooth at the branes, with the scale factor reaching its minimum value, we must have  $\dot{a}(\tau_i) = 0$ .
5. To maintain a perturbative analysis throughout the cosmological evolution, the dilaton field must be bounded from above,  $\phi(\tau) \leq \phi_c$ , with  $g_c = e^{\phi_c}$  sufficiently small.

Using the state equation (4.1) and the entropy conservation  $(\mathbf{s})'$ , valid throughout the cosmological evolution, we obtain

$$aT = a(\tau_i) T_c \equiv \gamma_{\infty}, \quad (i = 1, \dots, n). \quad (4.2)$$



The dilaton equation **(iii)'** can be easily integrated since  $\rho = (d - 1)P$  in each interval, yielding

$$\dot{\phi} = c_i^- \frac{e^{2\phi} N}{a^{d-1}} \quad \text{for } \tau_{i-1} < \tau < \tau_i, \quad \dot{\phi} = c_i^+ \frac{e^{2\phi} N}{a^{d-1}} \quad \text{for } \tau_i < \tau < \tau_{i+1}, \quad (4.3)$$

where  $c_i^\pm = c_{i+1}^\mp$  are constants related to the tensions (3.32):

$$\kappa_i = 2 (c_i^- - c_i^+) \frac{e^{2\phi(\tau_i)}}{a(\tau_i)^{d-1}}. \quad (4.4)$$

To count the number of independent integration constants parametrizing the solution, we consider three independent equations, namely the  $N$ -equation **(i)**, the dilation equation **(iii)'** and the entropy conservation relation **(s)'**. These are second order in  $\phi$  and first order in  $a$ , leading to three integration constants in each branch. When we glue two solutions at  $\tau_i$ , the continuity of  $a$  and  $\phi$  across the brane, together with  $\dot{a}(\tau_{i+}) = \dot{a}(\tau_{i-}) = 0$  have to be imposed. Consequently, we are left with  $2 \times 3 - 4 = 2$  arbitrary integration constants.

In the following, we display solutions characterized by a single phase transition occurring at  $\sigma(\tau_c) = 0$ ,  $\tau_c \equiv 0$ . These solutions are obtained under the assumption that  $T_c$  is reached only once at  $\tau = \tau_c = 0$ . For space-time dimensions  $d > 2$ , the isotropic and homogeneous solutions may or may not have a non-trivial spatial curvature  $k$ . We display solutions in both cases.

## 4.1 Bouncing cosmology with vanishing curvature, $k = 0$

We first consider the case of vanishing spatial curvature ( $k = 0$ ), looking for a cosmological evolution satisfying the consistency conditions 1 to 5 listed above. As explained at the end of the previous subsection, the complete cosmological evolution is expected to depend on two arbitrary integration constants, which we may choose to be the values of the scale factor  $a_c$  and the dilaton field  $\phi_c$  at the brane. We find that the complete cosmological evolution in the sigma-model frame and in the conformal gauge,

$$\ln \frac{N}{a_c} = \ln \frac{a}{a_c} = \ln \frac{T_c}{T} = |\sigma|, \quad (4.5)$$

takes a simple form

$$\begin{aligned}\sigma &= \frac{\text{sign}(\tau)}{d-2} \left[ \eta_+ \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_+} \right) - \eta_- \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_-} \right) \right], \\ \phi &= \phi_c + \frac{\sqrt{d-1}}{2} \left[ \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_+} \right) - \ln \left( 1 + \frac{\omega a_c |\tau|}{\eta_-} \right) \right],\end{aligned}\tag{4.6}$$

where  $\eta_{\pm} = \sqrt{d-1} \pm 1$ . The parameter  $\omega$  is proportional to the brane tension  $\kappa_R$ , which is responsible for the gluing:

$$\omega = \kappa_R \frac{d-2}{4\sqrt{d-1}}, \quad \kappa_R = 2\sqrt{2(d-1)} \sqrt{n^* \Sigma_d} T_c^{d/2} e^{\phi_c}.\tag{4.7}$$

Note that  $\kappa_R$  is of order  $\mathcal{O}(e^{\phi_c} = g_c)$ , which is larger than the naive  $\mathcal{O}(e^{2\phi_c} = g_c^2)$  expectation from Eq. (4.4). In the neighborhood of the brane ( $|\kappa_R a_c \tau| \ll 1$ ), the metric is by construction regular while the dilaton field shows a conical singularity:

$$\begin{aligned}\sigma &= \frac{\text{sign}(\tau)}{d-2} \left( \frac{1}{\eta_-} - \frac{1}{\eta_+} \right) (\omega a_c \tau)^2 + \mathcal{O}(|\omega a_c \tau|^3) = \frac{\text{sign}(\tau)}{16(d-1)} (\kappa_R a_c \tau)^2 + \mathcal{O}(|\kappa_R a_c \tau|^3), \\ \phi &= \phi_c - \frac{\sqrt{d-1}}{2} \left( \frac{1}{\eta_-} - \frac{1}{\eta_+} \right) |\omega a_c \tau| + \mathcal{O}((\omega a_c \tau)^2) = \phi_c - \frac{|\kappa_R a_c \tau|}{4} + \mathcal{O}((\kappa_R a_c \tau)^2).\end{aligned}\tag{4.8}$$

The maximal value of the Ricci scalar (3.24) occurs at the bounce and it is given in terms of the brane tension by

$$\mathcal{R}_c = \frac{\kappa_R^2}{4} = \mathcal{O}(g_c^2).\tag{4.9}$$

Thus both higher derivative corrections and higher genus contributions in the effective action remain small throughout the evolution, and can be consistently neglected, provided that the critical value of the string coupling,  $g_c$ , is taken to be sufficiently small.

Far from the brane ( $|\kappa_R a_c \tau| \gg 1$ ), the dilaton is asymptotically constant, the temperature drops and the scale factor tends to infinity. The whole evolution in sigma-model frame describes a bounce, where the scale factor and temperature are smooth. In the Einstein frame however, they develop conical singularities at the brane inherited from the rescaling with the string coupling. The Einstein frame fields are given by

$$(N_E, a_E, 1/T_E) := e^{-\frac{2\phi}{d-2}} (N, a, 1/T).\tag{4.10}$$

At very early and late times ( $|\omega a_c \tau| \rightarrow +\infty$ ), the scaling properties of the dilaton motion and thermal contributions to the energy density in the Einstein frame and in the  $N_E =$

1 gauge are respectively  $1/a_E^{2(d-1)}$  and  $1/a_E^d$ . The whole cosmological evolution describes a bounce between two asymptotically radiation dominated Universes. The fact that in the Einstein frame the scale factor, the temperature and the dilaton bounce with conical singularities follows from the localized negative contribution of the brane to the pressure,  $P_B = -e^{-2\phi_c} \kappa_R \delta(\tau)$ . This localized pressure is not artificially added, but its origin lies in the stringy thermal duality properties of the system.

To complete our analysis we also display the cosmological evolution in two dimensions:

$$\begin{aligned}\sigma &= \frac{\text{sign}(\tau)}{2} \left[ \frac{\kappa_R a_c |\tau|}{2} - \ln \left( 1 + \frac{\kappa_R a_c |\tau|}{2} \right) \right], \\ \phi &= \phi_c - \frac{1}{2} \ln \left( 1 + \frac{\kappa_R a_c |\tau|}{2} \right).\end{aligned}\tag{4.11}$$

This solution was found in the context of the Hybrid models in Ref. [16], but it is also valid in the more general tachyon-free two-dimensional thermal models. Consistently, this solution is recovered when the space-time dimension  $d$  is formally treated as a real parameter, taking the limit  $d \rightarrow 2$  in Eqs (4.6) and (4.7).

For  $d > 2$ , figure 1 makes the comparison between (i) Classical General Relativity coupled to thermal radiation (dotted lines), (ii) Classical General Relativity coupled to thermal radiation and non-trivial motion for the dilaton field (dashed lines), and (iii) the thermal superstring indicating a phase transition at  $\tau_c = 0$  between the dual effective field theories (solid lines):

(i) In the first case, the scale factor,

$$a_{CGR}^{d-2} = a_c^{d-2} \frac{\eta_-}{\eta_+} (\omega a_c \tau + \eta_+ + \eta_-)^2,\tag{4.12}$$

develops the well known initial singularity at infinite temperature, here at  $\omega a_c \tau = -\eta_+ - \eta_-$ .

(ii) When the system is coupled to a non-trivial dilaton ( $\dot{\phi} \propto e^{2\phi} N/a^{d-1}$ ), the cosmological evolution takes the form (4.6), *without* the absolute values in the arguments. In this case,  $\sigma(\tau)$  vanishes and bounces at  $\tau = 0$ , so that  $\sigma(\tau) \geq 0$  is always satisfied and the whole evolution remains within the framework of the momentum effective field theory. In the Einstein frame, the scale factor and temperature are monotonic and,

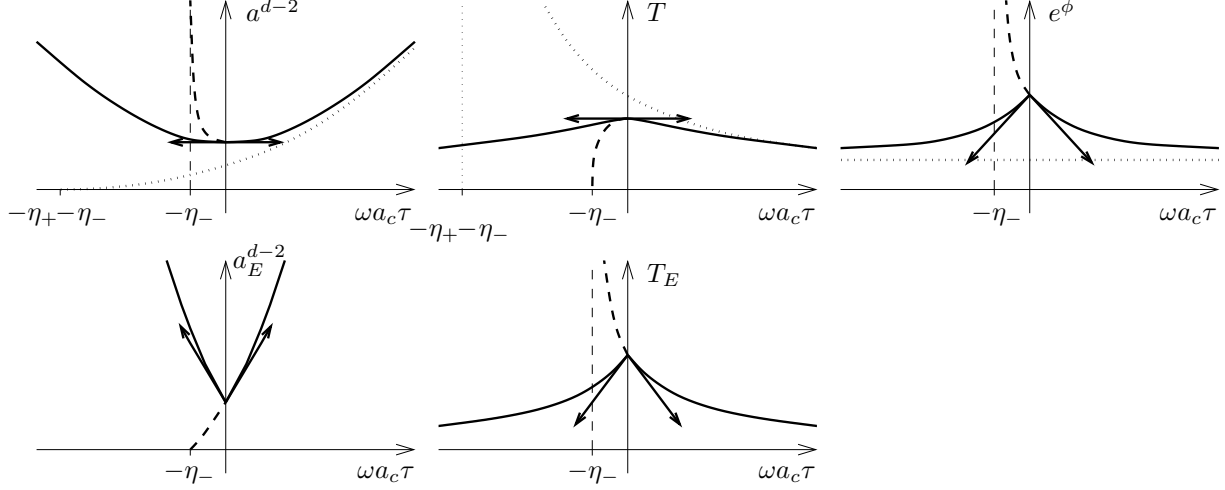


Figure 1: For  $k = 0$  and  $d > 2$ , comparison between: (i) Classical General Relativity coupled to radiation (dotted lines), (ii) Classical General Relativity coupled to radiation and dilaton motion (dashed lines), and (iii) the superstring picture (solid lines). Case (i) leads to the conventional initial curvature and infinite temperature singularities. In case (ii), the early times of the Universe are out of perturbative control. In case (iii), the winding  $\rightarrow$  momentum effective field theory phase transition at  $\tau = 0$  substitutes the previous non-perturbative regime with a pre-big bang cosmology at weak coupling. In Einstein frame, the whole evolution describes a bounce where the scale factor, the temperature and the dilaton develop conical singularities. Asymptotically, the two phases are radiation dominated.

when we go backward in time, the dilaton drives the Universe into an out of control non-perturbative regime ( $\phi \rightarrow +\infty$ ,  $a_E \rightarrow 0$ ,  $T_E \rightarrow +\infty$  when  $\omega a_c \tau \rightarrow -\eta_-$ ).

(iii) The phase transition at  $\tau = 0$  dictated by string theory effectively substitutes the previous non-perturbative regime in the momentum effective field theory with a perturbative pre-big bang cosmology in the dual winding effective field theory. In the Einstein frame, the scale factor and temperature satisfy:

$$a_E^{d-2} = a_c^{d-2} e^{-2\phi_c} \left( \frac{\omega a_c |\tau|}{\eta_+} + 1 \right) \left( \frac{\omega a_c |\tau|}{\eta_-} + 1 \right), \quad T_E = \frac{a_c T_c}{a_E}. \quad (4.13)$$

In two dimensions, the case (i) of Classical General Relativity coupled to thermal radiation only does not make sense. In the presence of dilaton motion, the comparison between cases (ii) and (iii) leads to conclusions identical to those in higher dimension.

## 4.2 Bouncing cosmology with non-vanishing curvature, $k \neq 0$

We now turn for  $d > 2$  to the case of a homogeneous and isotropic Universe with negative spatial curvature  $k = -1$ . As was the case of  $k = 0$ , the cosmological evolution is compatible

with the constraints 1–5 and with localized branes at  $\sigma(\tau_c) = 0$  triggering the transition between the winding-like and the momentum-like field theory. The scale factor  $a_c$  and dilaton  $\phi_c$  at the critical temperature  $T_c$  can be chosen as independent integration constants characterizing the whole solution. It is convenient to combine  $a_c$  and  $\phi_c$  in terms of a parameter  $\lambda$  which can be further used to define  $\alpha_{\pm}$ ,

$$\lambda := \frac{2}{d-2} n^* \Sigma_d e^{2\phi_c} a_c^2 T_c^d, \quad \alpha_{\pm} = \frac{\sqrt{d-2} \lambda \pm 2\sqrt{1+\lambda}}{2\sqrt{d-1}\sqrt{1+\lambda} + \sqrt{d-2}(2+\lambda)}, \quad (4.14)$$

satisfying  $1 > \alpha_+ > 0$  and  $\alpha_+ > \alpha_- > -1$ . These quantities appear explicitly in the algebraic expressions of the scale factor and the dilaton. In the conformal gauge (4.5),  $\sigma(\tau)$  and  $\phi(\tau)$  take the following form:

$$\begin{aligned} \sigma &= \frac{\text{sign}(\tau)}{d-2} \left[ \eta_+ \ln \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_- e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_-} \right) - \eta_- \ln \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_+ e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_+} \right) \right], \\ \phi &= \phi_c + \frac{\sqrt{d-1}}{2} \left[ \ln \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_- e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_-} \right) - \ln \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_+ e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_+} \right) \right]. \end{aligned} \quad (4.15)$$

For  $|\tau| \ll 1$ , the bounces for the metric and dilaton are respectively smooth and conical:

$$\begin{aligned} \sigma &= \frac{\text{sign}(\tau) (d-2) (2+\lambda)}{4} \tau^2 + \mathcal{O}(|\tau|^3), \\ \phi &= \phi_c - \frac{\sqrt{(d-1)(d-2)(1+\lambda)}}{2} |\tau| + \mathcal{O}(\tau^2). \end{aligned} \quad (4.16)$$

The brane tension  $\kappa_C$  is given in terms of  $a_c$ ,  $\lambda$  and the space-time dimension,

$$\kappa_C = \frac{2\sqrt{(d-1)(d-2)}}{a_c} \sqrt{1+\lambda}. \quad (4.17)$$

It determines the maximal value of the Ricci scalar, which is obtained at the bounce,

$$\mathcal{R}_c = \frac{\kappa_C^2}{4} = \mathcal{O}(a_c^{-2}) + \mathcal{O}(g_c^2). \quad (4.18)$$

The latter is small provided the inverse scale factor and string coupling at the transition are chosen sufficiently small. In this case, higher derivative terms and higher loop corrections can be consistently neglected throughout the evolution.

At very early and late times ( $|\tau| \gg 1$ ), the dilaton motion vanishes while the Universe cools and grows to infinity. In the Einstein frame, the scale factor  $a_E$ , the temperature  $T_E$

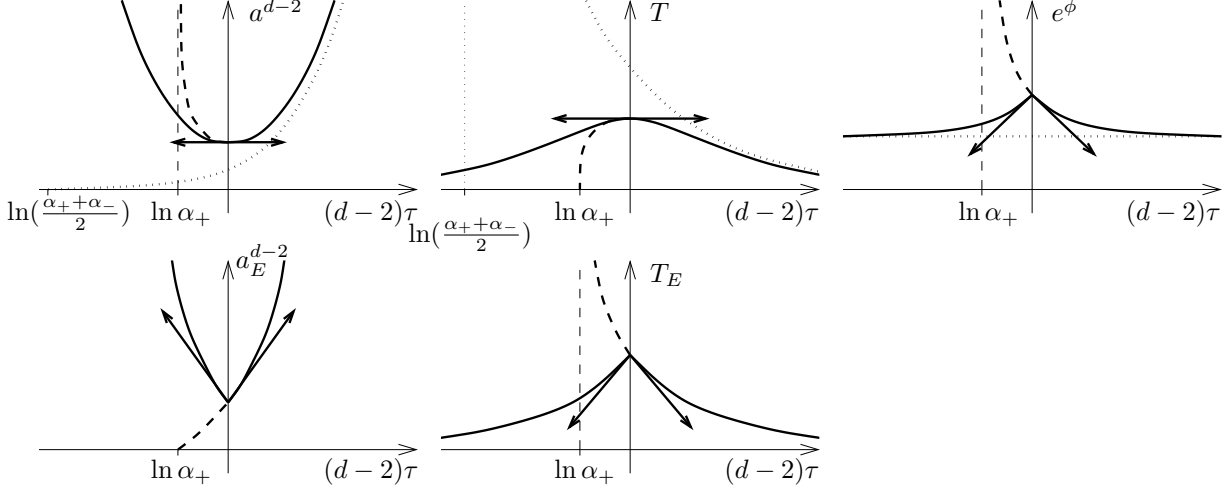


Figure 2: For  $k = -1$  and  $d > 2$ , comparison between: (i) Classical General Relativity coupled to radiation (dotted lines), (ii) Classical General Relativity coupled to radiation and dilaton motion (dashed lines), and (iii) the superstring picture (solid lines). Case (i) leads to the conventional initial curvature and infinite temperature singularities. In case (ii), the early times of the Universe are out of perturbative control. In case (iii), the winding  $\rightarrow$  momentum effective field theory phase transition at  $\tau = 0$  substitutes the previous non-perturbative regime with a pre-big bang cosmology at weak coupling. In Einstein frame, the whole evolution describes a bounce where the scale factor, the temperature and the dilaton develop conical singularities. Asymptotically, the two phases are curvature dominated.

and the dilaton bounce with conical behavior at the origin  $\tau = 0$ . In the asymptotic limits  $\tau \rightarrow \pm\infty$ , the dilaton motion, the thermal radiation and the spatial curvature contribute to the energy density as  $1/a_E^{2(d-1)}$ ,  $1/a_E^d$  and  $1/a_E^2$  (in the gauge  $N_E = 1$ ). As a result, the cosmological solution describes a bounce between two asymptotically curvature dominated Universes. As a result, the cosmological solution describes a bounce between two asymptotically curvature dominated Universes.

Figure 2 makes the comparison between (i) Classical General Relativity coupled to thermal radiation (dotted lines), (ii) Classical General Relativity coupled to thermal radiation and non-trivial motion for the dilaton field (dashed lines), and (iii) the superstring cosmology where two dual effective field theories are connected by a phase transition at  $\tau_c = 0$  (solid lines). The conclusions are qualitatively similar to those described for  $k = 0$ :

- (i) In the first case, the infinite curvature and temperature singularities occur at the initial time  $(d-2)\tau = \ln(\frac{\alpha_+ + \alpha_-}{2})$ ,

$$a_{CGR}^{d-2} = a_c^{d-2} \frac{(1 - \alpha_+)^{\eta_-}}{(1 - \alpha_-)^{\eta_+}} \left( e^{\frac{d-2}{2}\tau} - \frac{\alpha_+ + \alpha_-}{2} e^{-\frac{d-2}{2}\tau} \right)^2. \quad (4.19)$$

- (ii) In presence of dilaton motion ( $\dot{\phi} \propto e^{2\phi} N/a^{d-1}$ ), the cosmological evolution given in

(4.15) with no absolute values for  $\tau$  behaves as follows.  $\sigma(\tau)$  vanishes and bounces at  $\tau = 0$ , which implies the evolution is described in the momentum effective field theory only. The dilaton, together with the Einstein frame scale factor and temperature are monotonic. Thus, the early times of the Universe are out of perturbative control, ( $\phi \rightarrow +\infty$ ,  $a_E \rightarrow 0$ ,  $T_E \rightarrow +\infty$  when  $(d-2)\tau \rightarrow \ln \alpha_+$ ).

(iii) In the thermal superstring case, the phase transition at  $\tau = 0$  replaces the above non-perturbative regime with a perturbative pre-big bang cosmology in the dual winding effective field theory. In the Einstein frame, the scale factor and temperature are

$$a_E^{d-2} = a_c^{d-2} e^{-2\phi_c} \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_+ e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_+} \right) \left( \frac{e^{\frac{d-2}{2}|\tau|} - \alpha_- e^{-\frac{d-2}{2}|\tau|}}{1 - \alpha_-} \right), \quad T_E = \frac{a_c T_c}{a_E}. \quad (4.20)$$

The case  $k = -1$  gives us the possibility to consider the evolution of the Universe at genus-0, *i.e.* without the genus-1 thermal corrections. Indeed, setting  $\lambda=0$  in Eq. (4.17), the brane tension at the tree level approximation does not vanish,

$$\kappa_C^{\text{tr}} = \frac{2\sqrt{(d-1)(d-2)}}{a_c}. \quad (4.21)$$

This fact indicates a non-trivial dilaton motion still exists at this order, provided  $d > 2$ . The explicit solution at this approximation is obtained from Eq. (4.15) in the limit  $\lambda \rightarrow 0$  and can be brought into the form,

$$\begin{aligned} \sigma &= \frac{\text{sign}(\tau)}{d-2} \left[ \eta_+ \ln \left( \frac{\cosh \left( \frac{d-2}{2} (|\tau| + \tau^{\text{tr}}) \right)}{\cosh \left( \frac{d-2}{2} \tau^{\text{tr}} \right)} \right) - \eta_- \ln \left( \frac{\sinh \left( \frac{d-2}{2} (|\tau| + \tau^{\text{tr}}) \right)}{\sinh \left( \frac{d-2}{2} \tau^{\text{tr}} \right)} \right) \right], \\ \phi &= \phi_c - \frac{\sqrt{d-1}}{2} \left[ \ln \left( \frac{\sinh \left( \frac{d-2}{2} (|\tau| + \tau^{\text{tr}}) \right)}{\sinh \left( \frac{d-2}{2} \tau^{\text{tr}} \right)} \right) - \ln \left( \frac{\cosh \left( \frac{d-2}{2} (|\tau| + \tau^{\text{tr}}) \right)}{\cosh \left( \frac{d-2}{2} \tau^{\text{tr}} \right)} \right) \right], \end{aligned} \quad (4.22)$$

where we have defined

$$\tau^{\text{tr}} = \frac{\ln (\sqrt{d-1} + \sqrt{d-2})}{d-2}. \quad (4.23)$$

This evolution does not describe flat (Minkowski) space, as is the case of vanishing spatial curvature. Instead, it corresponds to a dual time-dependent curvature dominated cosmology which bounces at  $\tau = 0$ , when the spacelike brane is crossed. The flat (Minkowski) space and the above  $k = -1$  tree level cosmology are dual, as follows from the fact that they are both solutions at genus-0 without central charge deficit ( $\delta c = 0$ ) of the underlying conformal worldsheet theory.

Given the fact that the notion of spatial curvature  $k$  is irrelevant in two dimensions, it is a non-trivial consistency check to recover the two-dimensional solution (4.11) by treating the space-time dimension as a continuous parameter and taking the limit  $d \rightarrow 2$  in (4.15). In particular,  $\kappa_C \sim \kappa_R$  when  $d \rightarrow 2$ . Since at genus-1 both thermal evolutions with  $k = 0$  and  $k = -1$  tend to the same solution when  $d \rightarrow 2$ , one expects the “dual-to-flat” classical cosmology (4.22) to converge to the flat (Minkowski) genus-0 phase transition in this limit. This is easily checked, which shows that the weak coupling limit  $\phi_c \rightarrow -\infty$  and the  $d \rightarrow 2$  limit are commuting.

## 5 Conclusions

The scope of this work was to establish a stringy mechanism able to resolve both the Hagedorn instabilities of finite temperature superstring theory as well as the initial curvature singularity of the induced cosmology in arbitrary dimensions. The key ingredients of this mechanism were first isolated in the context of the two-dimensional Hybrid models whose right-moving sector enjoys the *MSDS* structure. The latter ensures boson/fermion degeneracy in the right-moving massive level.

In this paper we have shown that these stringy ingredients are generic in a large class of  $\mathcal{N}_4 = (4, 0)$  superstring models. Tachyon-free, thermal configurations in  $d$ -dimensional  $\mathcal{N}_4 = (4, 0)$  models can be constructed in the presence of special “gravito-magnetic” fluxes. The fluxes modify the thermal vacuum by injecting into it non-trivial momentum and winding charges, lifting the Hagedorn instabilities of the canonical thermal ensemble. The key property is the restoration of the thermal T-duality symmetry of the stringy thermal system, implying a maximal critical temperature. In all such models there are three characteristic regimes, each with a distinct effective field theory description: Two dual asymptotically cold regimes associated with the light thermal momentum and light thermal winding states, and the intermediate regime where additional massless thermal states appear leading to enhanced Euclidean gauge symmetry.

Taking into account the genus-0 backgrounds associated with the extra massless states, we have shown that they source Euclidean branes localized at the critical point, which glue the two asymptotic momentum and winding regimes. By utilizing string calculational techniques,



we were able to establish that the thermal partition function can be well-approximated by that of massless thermal radiation up to the critical temperature. The partition function exhibits a conical structure as a function of the thermal modulus  $\sigma$ , generalizing the two dimensional Hybrid result to any dimension. In all regimes, the equation of state is effectively given by

$$\rho = (d-1)P = (d-1)n^*\Sigma_d T_c^d e^{-d|\sigma|},$$

modulo exponentially suppressed contributions. Both the energy density and pressure are bounded from above attaining their maximal values at the critical temperature. We show explicitly that the conical structure is resolved by the spacelike branes, which occur when  $\sigma = 0$ . These branes provide a localized negative pressure which is crucial in evading the constraints on realizing singularity-free, bouncing cosmologies, imposed by the singularity theorems of classical general relativity [1]. Utilizing the above ingredients, we were able to obtain a string effective Lorentzian action covering the three characteristic regimes simultaneously. This action incorporates the spacelike brane that glues together the winding and the momentum regimes.

Taking into account the localized negative pressure contribution from the branes, as well as the bulk thermal corrections, we have obtained *non-singular* analytic cosmological solutions, describing bouncing thermal and curvature dominated Universes. The bounce occurs at the phase transition between the momentum and winding regimes. The cosmological solutions remain perturbative throughout the evolution, provided that the value of the string coupling at the branes is sufficiently small. These bouncing cosmologies are the first higher dimensional examples, where both the Hagedorn singularities as well as the classical Big-Bang singularity are successfully resolved, remaining perturbative throughout the evolution.

In this work we presented cosmological solutions associated with a single phase transition, with the spacelike branes occurring at a specific point in time. The brane interpretation leads us to believe that consistent cosmological solutions exist associated with a series of phase transitions, described by a distribution of branes localized in several points in time. At each brane the temperature reaches the maximal value. This work is currently under investigation [29].

Having in our disposal exact cosmological solutions, we can explicitly calculate the spectrum of fluctuations at early times, say at the time locations of the branes, determine their

propagation at latter cosmological times and compare them to the current and future observational data. This is possible since we have analytical control on the theory describing the brane. This work is currently in progress [30].

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## References

- [1] R. Penrose, “Structure of space-time,” in *Battelle Rencontres, 1967, Lectures in Mathematics and Physics*, edited by C.M. DeWitt and J.A. Wheeler, pp. 121-235, Benjamin, New York, 1968.
- S. W. Hawking and G. F. R. Ellis, “The large scale structure of space-time,” Cambridge University Press, Cambridge, 1973.
- S. Hawking and G. F. R. Ellis, “Singularities in homogeneous world models,” *Phys. Lett.* **17** (1965) 246.
- [2] E. Kiritsis and C. Kounnas, “Dynamical topology change, compactification and waves in a stringy early universe,” arXiv:hep-th/9407005.
- E. Kiritsis and C. Kounnas, “Dynamical topology change in string theory,” *Phys. Lett. B* **331** (1994) 51 [arXiv:hep-th/9404092].
- E. Kiritsis and C. Kounnas, “Dynamical topology change, compactification and waves in string cosmology,” *Nucl. Phys. Proc. Suppl.* **41** (1995) 311 [arXiv:gr-qc/9701005].

- E. Kiritsis and C. Kounnas, “String gravity and cosmology: Some new ideas,” arXiv:gr-qc/9509017.
- [3] R. H. Brandenberger and C. Vafa, “Superstrings in the early universe,” Nucl. Phys. B **316** (1989) 391.
- G. Veneziano, “Scale factor duality for classical and quantum strings,” Phys. Lett. B **265** (1991) 287.
- A. Tseytlin and C. Vafa, “Elements of string cosmology,” Nucl. Phys. B **372** (1992) 443 [arXiv:hep-th/9109048].
- [4] M. Gasperini and G. Veneziano, “Pre-big bang in string cosmology,” Astropart. Phys. **1** (1993) 317 [arXiv:hep-th/9211021].
- M. Gasperini, M. Maggiore and G. Veneziano, “Towards a nonsingular pre - big bang cosmology,” Nucl. Phys. B **494** (1997) 315 [arXiv:hep-th/9611039].
- M. Gasperini and G. Veneziano, “Singularity and exit problems in two-dimensional string cosmology,” Phys. Lett. B **387** (1996) 715 [arXiv:hep-th/9607126].
- R. Brustein, M. Gasperini and G. Veneziano, “Duality in cosmological perturbation theory,” Phys. Lett. B **431** (1998) 277 [arXiv:hep-th/9803018].
- M. Gasperini and G. Veneziano, “The pre - big bang scenario in string cosmology,” Phys. Rept. **373** (2003) 1 [arXiv:hep-th/0207130].
- [5] J. J. Atick and E. Witten, “The Hagedorn transition and the number of degrees of freedom of string theory,” Nucl. Phys. B **310** (1988) 291.
- [6] C. Kounnas and B. Rostand, “Coordinate dependent compactifications and discrete symmetries,” Nucl. Phys. B **341** (1990) 641.
- [7] I. Antoniadis and C. Kounnas, “Superstring phase transition at high temperature,” Phys. Lett. B **261** (1991) 369.
- I. Antoniadis, J. P. Derendinger and C. Kounnas, “Nonperturbative temperature instabilities in  $\mathcal{N} = 4$  strings,” Nucl. Phys. B **551** (1999) 41 [arXiv:hep-th/9902032].
- C. Kounnas, “Universal thermal instabilities and the high-temperature phase of the  $\mathcal{N} = 4$  superstrings,” arXiv:hep-th/9902072.

- [8] J. L. F. Barbon and E. Rabinovici, “Touring the Hagedorn ridge,” arXiv:hep-th/0407236.
- [9] J. L. Davis, F. Larsen and N. Seiberg, “Heterotic strings in two dimensions and new stringy phase transitions,” JHEP **0508** (2005) 035 [arXiv:hep-th/0505081].  
 N. Seiberg, “Long strings, anomaly cancellation, phase transitions, T-duality and locality in the 2d heterotic string,” JHEP **0601** (2006) 057 [arXiv:hep-th/0511220].  
 J. L. Davis, “The moduli space and phase structure of heterotic strings in two dimensions,” Phys. Rev. D **74** (2006) 026004 [arXiv:hep-th/0511298].
- [10] S. Chaudhuri, “Finite temperature bosonic closed strings: Thermal duality and the Kosterlitz-Thouless transition,” Phys. Rev. D **65** (2002) 066008 [arXiv:hep-th/0105110].
- [11] K. R. Dienes and M. Lennek, “Adventures in thermal duality (I): Extracting closed form solutions for finite temperature effective potentials in string theory,” Phys. Rev. D **70** (2004) 126005 [arXiv:hep-th/0312216].  
 K. R. Dienes and M. Lennek, “Adventures in thermal duality (II): Towards a duality covariant string thermodynamics,” Phys. Rev. D **70** (2004) 126006 [arXiv:hep-th/0312217].
- [12] N. Matsuo, “Superstring thermodynamics and its application to cosmology,” Z. Phys. C **36** (1987) 289.  
 J. Kripfganz and H. Perl, “Cosmological impact of winding strings,” Class. Quant. Grav. **5** (1988) 453.  
 M. J. Bowick and S. B. Giddings, “High temperature strings,” Nucl. Phys. B **325** (1989) 631.  
 R. Easther, B. R. Greene, M. G. Jackson and D. N. Kabat, “String windings in the early universe,” JCAP **0502** (2005) 009 [arXiv:hep-th/0409121].  
 J. E. Lidsey, D. Wands and E. J. Copeland, “Superstring cosmology,” Phys. Rept. **337** (2000) 343 [arXiv:hep-th/9909061].  
 T. Battefeld and S. Watson, “String gas cosmology,” Rev. Mod. Phys. **78** (2006) 435 [arXiv:hep-th/0510022].

- N. Kaloper, L. Kofman, A. D. Linde and V. Mukhanov, “On the new string theory inspired mechanism of generation of cosmological perturbations,” JCAP **0610** (2006) 006 [arXiv:hep-th/0608200].
- N. Kaloper and S. Watson, “Geometric precipices in string cosmology,” Phys. Rev. D **77** (2008) 066002 [arXiv:0712.1820 [hep-th]].
- R. H. Brandenberger, “String gas cosmology,” arXiv:0808.0746 [hep-th].
- B. Greene, D. Kabat and S. Marnerides, “Bouncing and cyclic string gas cosmologies,” Phys. Rev. D **80** (2009) 063526 [arXiv:0809.1704 [hep-th]].
- [13] C. Angelantonj, C. Kounnas, H. Partouche and N. Toumbas, “Resolution of Hagedorn singularity in superstrings with gravito-magnetic fluxes,” Nucl. Phys. B **809** (2009) 291 [arXiv:0808.1357 [hep-th]].
- [14] C. Kounnas, “Massive boson-fermion degeneracy and the early structure of the universe,” Fortsch. Phys. **56** (2008) 1143 [arXiv:0808.1340 [hep-th]].
- I. Florakis and C. Kounnas, “Orbifold symmetry reductions of massive boson-fermion degeneracy,” Nucl. Phys. B **820** (2009) 237 [arXiv:0901.3055 [hep-th]].
- [15] I. Florakis, C. Kounnas and N. Toumbas, “Marginal deformations of vacua with massive boson-fermion degeneracy symmetry,” Nucl. Phys. B **834** (2010) 273 [arXiv:1002.2427 [hep-th]].
- [16] I. Florakis, C. Kounnas, H. Partouche and N. Toumbas, “Non-singular string cosmology in a 2d Hybrid model,” Nucl. Phys. B **844** (2011) 89 [arXiv:1008.5129 [hep-th]].
- [17] D. Kutasov and N. Seiberg, “Number of degrees of freedom, density of states and tachyons in string theory and CFT,” Nucl. Phys. B **358** (1991) 600.
- [18] K. Dienes, “Modular invariance, finiteness, and misaligned supersymmetry: New constraints on the numbers of physical string states,” Nucl. Phys. B **429** (1994) 533 [arXiv:hep-th/9402006].
- [19] J. Scherk and J. H. Schwarz, “Spontaneous breaking of supersymmetry through dimensional reduction,” Phys. Lett. B **82** (1979) 60.

- [20] R. Rohm, “Spontaneous supersymmetry breaking in supersymmetric string theories,” Nucl. Phys. B **237** (1984) 553.
- C. Kounnas and M. Porrati, “Spontaneous supersymmetry breaking in string theory,” Nucl. Phys. B **310** (1988) 355.
- S. Ferrara, C. Kounnas and M. Porrati, “ $\mathcal{N} = 1$  superstrings with spontaneously broken symmetries,” Phys. Lett. B **206** (1988) 25.
- S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, “Effective superhiggs and Str  $M^2$  from four-dimensional strings,” Phys. Lett. B **194** (1987) 366.
- S. Ferrara, C. Kounnas and M. Porrati, “Superstring solutions with spontaneously broken four-dimensional supersymmetry,” Nucl. Phys. B **304** (1988) 500.
- [21] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, “Superpotentials in IIA compactifications with general fluxes,” Nucl. Phys. B **715** (2005) 211 [arXiv:hep-th/0411276].
- J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, “Fluxes and gaugings:  $\mathcal{N} = 1$  effective superpotentials,” Fortsch. Phys. **53** (2005) 926 [arXiv:hep-th/0503229].
- L. Andrianopoli, M. A. Lledo and M. Trigiante, “The Scherk-Schwarz mechanism as a flux compactification with internal torsion,” JHEP **0505** (2005) 051 [arXiv:hep-th/0502083].
- G. Dall’Agata and N. Prezas, “Scherk-Schwarz reduction of M-theory on  $G_2$ -manifolds with fluxes,” JHEP **0510** (2005) 103 [arXiv:hep-th/0509052].
- [22] P. H. Ginsparg and C. Vafa, “Toroidal compactification of nonsupersymmetric heterotic strings,” Nucl. Phys. B **289** (1987) 414.
- V. P. Nair, A. D. Shapere, A. Strominger and F. Wilczek, “Compactification of the twisted heterotic string,” Nucl. Phys. B **287** (1987) 402.
- S. P. Patil and R. Brandenberger, “Radion stabilization by stringy effects in general relativity,” Phys. Rev. D **71** (2005) 103522 [arXiv:hep-th/0401037].
- [23] F. Bourliot, J. Estes, C. Kounnas and H. Partouche, “Cosmological phases of the string thermal effective potential,” Nucl. Phys. B **830** (2010) 330 [arXiv:0908.1881 [hep-th]].

- J. Estes, C. Kounnas and H. Partouche, “Superstring cosmology for  $\mathcal{N}_4 = 1 \rightarrow 0$  superstring vacua,” accepted by Fortsch. Phys., arXiv:1003.0471 [hep-th].
- J. Estes, L. Liu and H. Partouche, “Massless D-strings and moduli stabilization in type I cosmology,” JHEP **1106** (2011) 060 [arXiv:1102.5001 [hep-th]].
- F. Bourliot, J. Estes, C. Kounnas and H. Partouche, “Thermal and quantum induced early superstring cosmology,” arXiv:0910.2814 [hep-th].
- [24] T. Catelin-Jullien, C. Kounnas, H. Partouche and N. Toumbas, “Thermal/quantum effects and induced superstring cosmologies,” Nucl. Phys. B **797** (2008) 137 [arXiv:0710.3895 [hep-th]].
- T. Catelin-Jullien, C. Kounnas, H. Partouche and N. Toumbas, “Induced superstring cosmologies and moduli stabilization,” Nucl. Phys. B **820** (2009) 290 [arXiv:0901.0259 [hep-th]].
- F. Bourliot, C. Kounnas and H. Partouche, “Attraction to a radiation-like era in early superstring cosmologies,” Nucl. Phys. B **816** (2009) 227 [arXiv:0902.1892 [hep-th]].
- T. Catelin-Jullien, C. Kounnas, H. Partouche and N. Toumbas, “Thermal and quantum superstring cosmologies,” Fortsch. Phys. **56** (2008) 792 [arXiv:0803.2674 [hep-th]].
- H. Partouche, “Attractions to radiation-like eras in superstring cosmologies,” Fortsch. Phys. **58** (2010) 797 [arXiv:1003.0840 [hep-th]].
- [25] B. McClain and B. D. B. Roth, “Modular invariance for interacting bosonic strings at finite temperature,” Commun. Math. Phys. **111** (1987) 539.
- K. H. O’Brien and C. I. Tan, “Modular invariance of thermopartition function and global phase structure of heterotic string,” Phys. Rev. D **36** (1987) 1184.
- P. Ditsas and E. G. Floratos, “Finite temperature closed bosonic string in a finite volume,” Phys. Lett. B **201** (1988) 49.
- [26] G. Mostow, “Strong rigidity of locally symmetric spaces,” Annals of mathematics studies, **78**, Princeton University Press, Princeton 1973.
- G. Prasad, “Strong rigidity of Q-rank 1 lattices,” Inventiones Mathematicae **21** (1973), 255.

- [27] G. D. Starkman (ed.), “Topology of the universe,” Proceedings, Conference, Cleveland, USA, October 17-19, 1997.
- [28] N. Kaloper, J. March-Russell, G. D. Starkman and M. Trodden, “Compact hyperbolic extra dimensions: Branes, Kaluza-Klein modes and cosmology,” Phys. Rev. Lett. **85** (2000) 928 [hep-ph/0002001].
- [29] I. Florakis, C. Kounnas, H. Partouche, N. Toumbas and J. Troost, *Work in progress*.
- [30] R. Brandenberger, C. Kounnas, H. Partouche, S. Patil and N. Toumbas, *Work in progress*.